3.5 Global Analysis

3.5.1 Introduction

The 4-dimensional variational data assimilation (4D-Var) system for the JMA Global Spectral Model (GSM) has been in operation since February 2005 in place of the 3-dimensional variational data assimilation (3D-Var) system. The JMA global 4D-Var system is used for the global cycle analysis four times a day (00,06,12,18UTC) and the global early analysis for the GSM forecast and the typhoon model forecast (00,06,12,18UTC). The scheme has the following benefits over the 3D-Var scheme.

The dynamics and physics of the forecast model are considered in assimilating data. As a result, observational data are optimally used in a meteorologically consistent way, so that the analysis increments become flow-dependent (Fig.3.5.1).

The observations are assimilated at appropriate observation time.

It can directly assimilate all observations data including precipitation amount that can be derived from model variables.

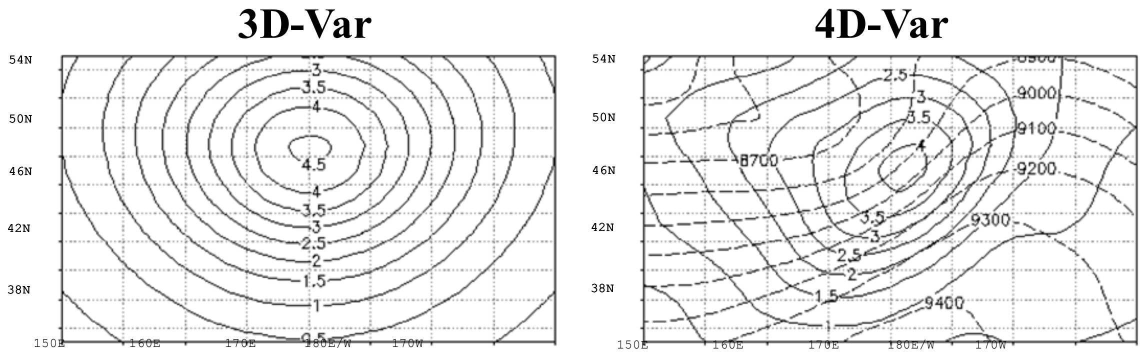

Fig. 3.5.1 Analysis increments (solid line) by 3D-Var (left) and 4D-Var (right) when one pseudo observation height data (departure 5m) is assimilated (the 20th model level). The broken line indicates the first guess height field. 4D-Var analysis increments are flow-dependent in accordance with the first guess field.

3.5.2 Description of the algorithm

The 4D-Var uses 3-9 hour forecast from GSM (TL319L40) as a first guess (background). All data within 3 hours from analysis time are assimilated at appropriate observation time with hourly assimilation slots, whereas they are regarded as observed at the analysis time in the 3D-Var. The cost-function measures the distance between the model trajectory (background) and the observations over a 6-hour assimilation window.

An incremental method (Courtier et al. 1994) is adopted in the 4D-Var to save computer resources. The method computes analysis increment at lower resolution (inner loop:T106L40) and then adds the increment to the high-resolution first guess (outer loop:TL319L40).

To obtain the analysis increment Δx_i, the minimization of the cost function J defined by Eq. (3.5.1) is performed in the inner loop.

$$
\begin{aligned}
&J(\Delta x_0) = \frac{1}{2}\Delta x_0{}^{\mathrm{T}}\,\mathrm{B}^{-1}\,\Delta x_0 + \frac{1}{2}\sum_{i=0}^{n}(\mathrm{H}_i\Delta x_i - d_i)^{\mathrm{T}}\,\mathrm{R}_i{}^{-1}(\mathrm{H}_i\Delta x_i - d_i) + J_C \\
&\Delta x_{i+1} = \mathrm{M}_i\Delta x_i = \mathrm{M}_i\mathrm{M}_{i-1}\mathrm{M}_{i-2}\cdots\mathrm{M}_0\mathrm{N}\Delta x_0
\end{aligned}
\qquad (3.5.1)
$$

where subscript i indicates the time and n denotes the end of the assimilation window. Δx_0 is the low resolution increment at the initial time before the initialization, and Δx_i is the increment evolved according to the tangent linear model from the initial time to time i and R_i denotes the covariance matrix of observation errors at time i and B is the covariance matrix of background errors, which are described in detail in sections 3.5.6 and 3.5.7. M_i is the tangent linear (TL) model of the low resolution Non-linear (NL) forecast model M_i described in detail in section 3.5.4. N is a nonlinear normal-mode initialization operator (Machenhauer 1977). H_i is the TL operator of the observation operator H_i. The innovation vector is given at each assimilation slot by $d_i = y_i^o - H_i x_i^b$, where x_i^b is the background state evolved by the high resolution NL model, and y_i^o is the observation data at time i. J_C is the penalty term to suppress the gravity wave described in section 3.5.5.

To minimize the cost function J, the limited memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) algorithm (Liu and Nocedal 1989) with Veerse's preconditioner (Veerse 2000) is applied. Here, the gradient of the cost function ∇J is required. It is obtained from the following adjoint procedures (3.5.2), which is computed reverse in time.

$$
\begin{aligned}
&p_{n+1} = 0 \\
&p_i = \mathrm{M}_i{}^{\mathrm{T}} p_{i+1} + \mathrm{H}_i{}^{\mathrm{T}}\mathrm{R}_i{}^{-1}(\mathrm{H}_i\Delta x_i - d_i) \qquad (i = n, \cdots, 1) \\
&p_0 = \mathrm{M}_0{}^{\mathrm{T}} p_1 + \mathrm{B}^{-1}(\Delta x_0) + \mathrm{H}_0{}^{\mathrm{T}}\mathrm{R}_0{}^{-1}(\mathrm{H}_i\Delta x_i - d_i) \\
&\nabla J(\Delta x_0) = p_0
\end{aligned}
\qquad (3.5.2)
$$

where p_i is a dummy variable, $\mathrm{M}_i^{\mathrm{T}}$ is the adjoint (AD) model of the TL model M_i, and $\mathrm{H}_i^{\mathrm{T}}$ is the AD operator of H_i.

The analyzed variables are the relative vorticity, divergence, temperature, surface pressure and the logarithm of specific humidity in the spectral space on the model layers (eta-coordinate). Observational data y_i^o are wind vector, temperature, relative humidity, satellite radiances, etc.

After getting the low resolution increment Δx_i, obtained from the minimization of the cost function in the inner loop, is interpolated to the high resolution analysis increment. By adding the increment to the first guess field, high resolution analysis field is derived.

3.5.3 Description of the procedure

In our system global analyses are performed 4 times (00/06/12/18UTC) a day. The observations within 3 hours after and before each analysis time (assimilation window) are assimilated. The flow of 4D-Var is shown in Fig. 3.5.2 for the case of 12UTC analysis time. It is the same for the cycle and early analyses.

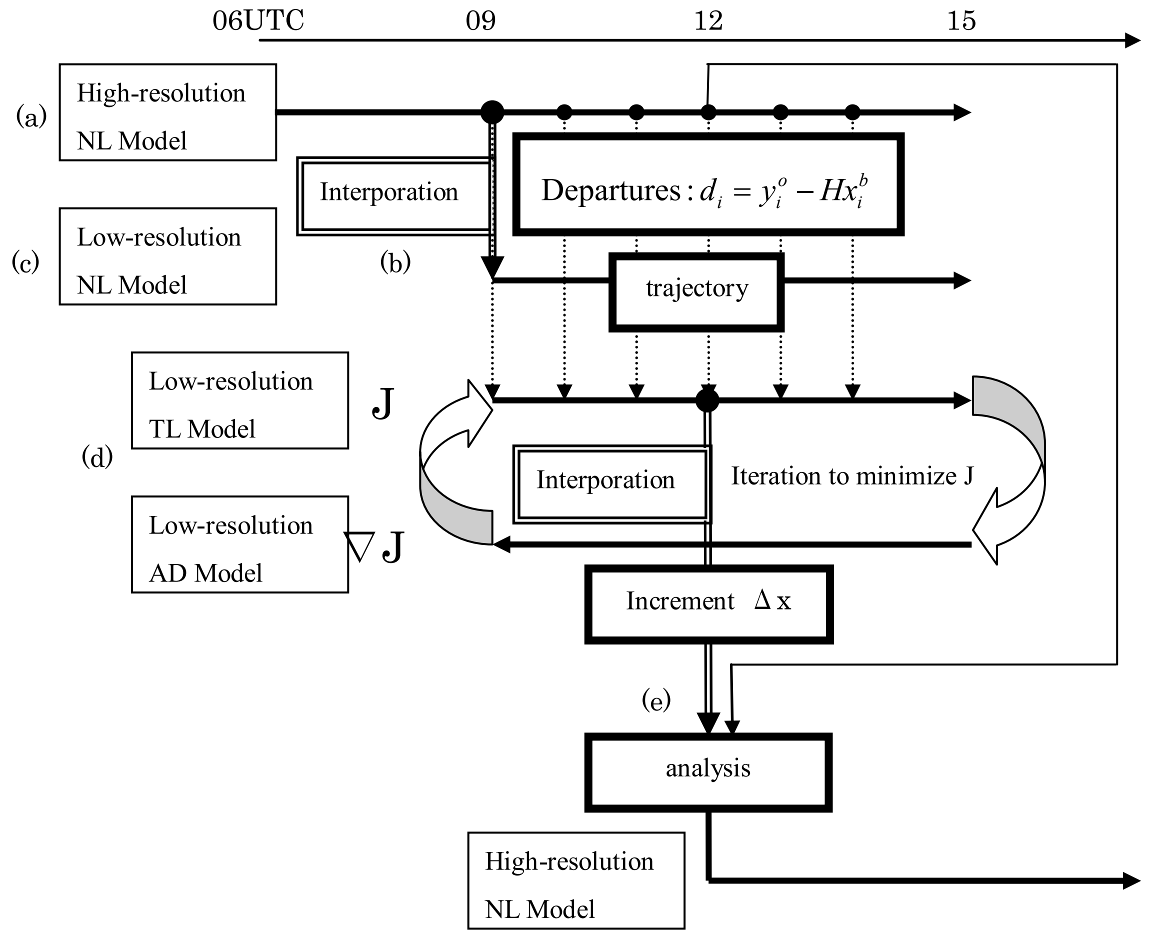

Fig. 3.5.2 Flow of 4D-Var procedure for the case of 12UTC analysis time

The procedure is as follows:

(a) 9-hour high resolution outer NL model forecast (06UTC-15UTC) of GSM from previous analysis is used as a first guess (background). The departures between the model trajectory and observations $d_i = y_i^o - H_i x_i^b$ over a 6-hour assimilation window (09UTC-15UTC) are measured. Observations are organized in six time-slots. The time intervals for the first and last slots are 0.5 hour and 1.5 hour and the others are 1 hour (Fig.3.5.3). All observations in each time slot are regarded as observed in each representative time. Then only 5-hour forecast is necessary in the 4D-Var, which contributes to the reduction of the computations.

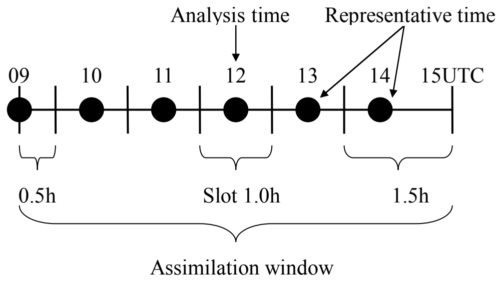

Fig. 3.5.3 Schematic diagram of time slots for the analysis time 12UTC. The black circles indicate the representative time of each time slot.

(b) The 3 hour forecast field (valid at 09UTC) of the high resolution model (TL319L40 GSM semi-Lagrangian model) is interpolated into the field of the low resolution inner model (T106L40 GSM Eulerian model) as an initial field. The interpolation is performed not only horizontally but also vertically to consider the difference of the topography between TL319 and T106, because the surface pressure is different so the pressure level is different. The vertical interpolation plays an essential role in running the T106 inner NL model, TL model and AD model (in 3D-Var it is not considered).
(c) The low resolution inner NL model (T106) is performed from the interpolated initial field to calculate the background state in the low resolution model space.
(d) The TL model and AD model with the innovation vector $d_i = y_i^o - Hx_i^b$ are performed to calculate the cost function J and its gradient ∇J. These processes are iterated to minimize the cost function J. The iteration is performed up to 70, which consists of 35 times with simple physics and 35 times with advanced physics. The background trajectory is not updated in our system.
(e) After the minimization of J, the field of 3 hour forecast (valid at 12UTC) of the TL model is chosen to be the analysis increment. It is interpolated horizontally and vertically for the high resolution model. Finally the analysis increment is added to the high resolution first guess field (valid at 12UTC) to obtain the final product.

3.5.4 TL model

The inner TL model is basically based on the inner NL model, the JMA GSM in 2001 (GSM0103; JMA 2002), which is an Eulerian model though the outer NL model is a semi-Lagrangian model. Thus, the dynamics of the inner NL and TL models are different from those of the outer model. Because the inner model is the Eulerean model, the time step of integration depends on the maximum wind speed. Therefore, we set the time limit of the iteration of 4D-Var (currently 15 minutes) in the case of strong winds.

The TL model includes only simple physical processes based on GSM0103. In the first half of the 70 iterations, simplified physics is applied, which includes (a) initialization , (b) horizontal diffusion, (c) surface turbulent fluxes and (d) vertical turbulent diffusion. In the last half of the iterations advanced physics is applied, which includes all of the following processes.

(a) Initialization

To control the gravity wave nonlinear normal-mode initialization is adopted (Machenhauer 1977).

(b) Horizontal diffusion

Horizontal diffusion is set to be stronger than the original nonlinear GSM0103 based on Buizza (1998).

(c) Surface turbulent fluxes

The surface turbulent fluxes are formulated as the Monin-Obukhov bulk formulae based on GSM0103. The sensible and latent heat flux are perturbed only over the sea.

(d) Vertical turbulent diffusion

The level 2 turbulence closure scheme by Mellor and Yamada (1974) is used to represent the vertical diffusion of momentum, heat and moisture based on the local-K theory based on GSM0103. The diffusion coefficients are not perturbed.

(e) Gravity wave drag

The parameterization for the orographic gravity wave drag consists of two components: one for long waves (wavelength >100km) and the other for short waves (wavelength ≈ 10 km) based on GSM0103. The Richardson number is not perturbed in some part for the long waves which are assumed to propagate upward until reaching wave-breaking levels mainly in the stratosphere and exert drag there to avoid the unstable calculation during the iterations.

(f) Long-wave radiation

Two kinds of the long-wave radiation are included in the TL model. One is based on Mahfouf (1999). The tendency of the perturbed temperature T' is given by

$$\frac{\partial T'}{\partial t} = -\alpha \frac{g}{C_p} \frac{\partial}{\partial p} \left(4F\,T'/T\right),$$

where $\alpha = \dfrac{1}{1+\left(P_r/P\right)^{10}}, p_r = 300hPa$ and F is the net radiation fluxes, g and C_p denote the gravitational constant and the specific heat at constant pressure respectively. The other is related to the difference between the surface ground temperature T_G and the bottom level temperature T_1. The tendency of the temperature T for the non-linear model is:

$$\frac{\partial T}{\partial t} = const + \beta\left(T_G^4 - T_1^4\right),$$

where β is a coefficient given by a function of the position. The tendency of the perturbed temperature T' is given by:

$$\frac{\partial T'}{\partial t} = -4\beta T_1^3 T_1' \,.$$

(g) Clouds and large-scale precipitation

Clouds and large-scale precipitation are based on GSM0103. In GSM0103, clouds are prognostically determined in a similar fashion to that of Smith (1990). A simple statistical approach proposed by Sommeria and Deardorff (1977) is employed to compute the cloud amount and the cloud water content. The parameterization of the conversion rate from cloud ice to precipitation follows the scheme proposed by Sundqvist (1978). They are much simplified in the TL model. The cloud fraction, the amount of dropping cloud water, and the dependence on the water vapor of the isobaric specific heat are not perturbed. Only some variables are perturbed in computing the conversion from cloud water to precipitation and in computing the evaporation of the precipitation.

(h) Cumulus convection

Cumulus convection is based on GSM0103, prognostic Arakawa - Schubert scheme (1974), but much simplified. The vertical wind shear and the planetary mixing length are not perturbed. The magnitude of perturbation of mass-flux is set bound to stabilize the calculation, so it is not exactly linear.

3.5.5 Penalty term

The penalty term J_C, which is the third term of Eq. (3.5.1), is given by

$$J_C = a\left(\left|\mathrm{N_G}\Delta x_0\right|^2 + \sum_{i=1}^{\mathrm{maxslot}} \left|\mathrm{N_G}\Delta x_i\right|^2 \right) \tag{3.5.3}$$

where $\mathrm{N_G}$ denotes an operator to calculate the tendency of the gravity wave mode based on Machenhauer (1977). Δx_0 is the increment at the initial time before the initialization, and Δx_i is the increment evolved according to the tangent linear model from the initial time to the representative time of the i th time slot after the initialization and the summation is from the first time slot (i=1) to the last time slot (i=maxslot). a is a constant $3.0\times10^{-2}[\mathrm{s}^4/\mathrm{m}^2]$, determined empirically. Though this penalty term is introduced to suppress the gravity wave in the increment Δx_i, it is also effective to stabilize the calculation.

3.5.6 Background term

The background term, which is the first term of Eq. (3.5.1), dominates how the 4D-Var analysis procedure converts the difference between the observation data and first guess into corrections to the first guess. The multivariate couplings in the analysis variables are based on the geostrophic linear balance between mass and wind. To reduce the correlations among the analysis variables, control variables are introduced. In the algorithm some additional statistical relations are also considered such as the less geostrophic balance in the smaller horizontal and vertical scales, virtually no geostrophic balance near the equator, the dependency of the geostrophy on the vertical level, a weak coupling between divergence and vorticity, as well as between divergence and mass.

The control variables in the 4D-Var are the relative vorticity ς, unbalanced divergence D_U, unbalanced temperature and surface pressure $(T, Ps)_U$, the logarithm of specific humidity $\ln q$ in the spectral space on the model layers. Autocovariances of the control variables are assumed to be homogeneous and isotropic. The correlation structures do not depend on the geographical location, but vertical correlations depend on horizontal scale. The unbalanced variables D_U and $(T, Ps)_U$ are defined as

$$\begin{aligned} &\Delta D_U \equiv \Delta D - P\Delta\phi_B \qquad (\phi_B = \phi_B(\zeta)) \\ &\begin{pmatrix} \Delta T \\ \Delta p_S \end{pmatrix}_U \equiv \begin{pmatrix} \Delta T \\ \Delta p_S \end{pmatrix} - Q\Delta\phi_B - R\Delta D_U \end{aligned} \tag{3.5.4}$$

where P, Q, R are regression coefficients, ϕ_B is a modified mass variable derived from relative vorticity described as follows. Δ denotes the deviation from the first guess. This formulation is similar to that of ECMWF, they call the regression coefficients as the balance operator. The regression coefficients are computed statistically using the NMC method (Parrish and Derber 1992) with 24/48-hour forecast differences to estimate the total covariances for each total spectral coefficient.

In the following paragraphs, (a) modified balance mass variable, (b) the regression coefficients, (c)(d) the covariance matrix of background errors, and (e) conversions from control variables to analysis variables are described

(a) Modified balance mass variable

The geostrophic balance is well kept at midlevels in the troposphere in extratropics. In other areas the balance is weak. To consider these relationships a modified balance mass variable is introduced. The statistical relationships among relative vorticity, divergence and temperature and surface pressure are calculated. First, the singular value decomposition of the linear balance operator L[i] is conducted.

$$\Delta\widetilde{\phi}_B = L\Delta\zeta = UWV^T\Delta\zeta \tag{3.5.5}$$

where $\widetilde{\phi}_B$ is the original balance mass variable, W is a positive semi-definite diagonal matrix, U and V are orthogonal matrices. The decomposed modes depend on latitude: a singular mode with a small singular value has large amplitude in low latitude. Second, the regression coefficients between mass variables[ii], derived from temperature and

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[i] Each wave number components of L is denoted as

$$\delta\widetilde{\phi}_{Bn}^{\,m} = c_n^m \delta\zeta_{n-1}^m + c_{n+1}^m \delta\zeta_{n+1}^m \quad ((n,m) \neq (0,0), n = m, m+1, \cdots, N), \quad c_n^m = -\frac{2\Omega a^2}{n^2}\sqrt{\frac{n^2 - m^2}{4n^2 - 1}}, \qquad \delta\widetilde{\phi}_{B0}^{\,0} = 0$$

where Ω is angular velocity of the Earth, a Earth radius, n total wavenumber, m zonal wavenumber.

[ii] The mass variable Φ_k on the k-th model level is defined by

$$\Phi_k = \phi_k + R_d \overline{T_k} \ln p_k$$

surface pressure, and balance mass variables are calculated as follows:

$$D_n = \frac{\left\langle \left(U^T \Delta\Phi\right)_n^m \left(U^T \Delta\tilde{\phi}_B\right)_n^m \right\rangle}{\left\langle \left[\left(U^T \Delta\tilde{\phi}_B\right)_n^m\right]^2 \right\rangle} \tag{3.5.6}$$

where $\langle\ \rangle$, D_n and n denote the statistical mean and zonal wavenumber mean, a positive definite diagonal matrix and index of singular vectors in latitudinal wave number respectively. The regression coefficients D_n (0~1) indicates how much the geostrophic balance is satisfied. D_n is averaged over latitudinal wave numbers and vertical levels. Then the modified balance mass variables are constructed as follows;

$$\Delta\phi_B = UDU^T \Delta\tilde{\phi}_B = UDWV^T \Delta\zeta = \tilde{L}\Delta\zeta \tag{3.5.7}$$

Note that the modified balance operator $\tilde{L}$ consists of 1) the conversion from the spectral space to the singular vector space, 2) the product of the regression coefficients D, and 3) the conversion from the singular vector space to the spectral space. The correlation between the modified mass variables and unbalanced mass variables (i.e. original mass variables – modified balance mass variables) could be neglected in all regions including the tropics.

(b) Regression coefficients

The regression coefficient matrices P, Q, and R are calculated for each total wavenumber n as follows:

$$\begin{aligned}
P_n &= \left\langle \Delta D_n^m \left(\Delta\phi_{Bn}^{\ m}\right)^T \right\rangle \left\langle \Delta\phi_{Bn}^{\ m} \left(\Delta\phi_{Bn}^{\ m}\right)^T \right\rangle^{-1}, \\
Q_n &= \left\langle \begin{pmatrix} \Delta T_n^m \\ \Delta p_{sn}^{\ m} \end{pmatrix} \left(\Delta\phi_{Bn}^{\ m}\right)^T \right\rangle \left\langle \Delta\phi_{Bn}^{\ m} \left(\Delta\phi_{Bn}^{\ m}\right)^T \right\rangle^{-1}, \\
R_n &= \left\langle \left[\begin{pmatrix} \Delta T_n^m \\ \Delta p_{sn}^{\ m} \end{pmatrix} - Q_n \Delta\phi_{Bn}^{\ m} \right] \left(\Delta D_{Un}^{\ m}\right)^T \right\rangle \left\langle \Delta D_{Un}^{\ m} \left(\Delta D_{Un}^{\ m}\right)^T \right\rangle^{-1}
\end{aligned} \tag{3.5.8}$$

(c) Background error covariance matrix

The background error covariance matrices of the control variables are calculated for each wavenumber (n,m) and the matrix size is equivalent to the number of vertical levels.

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where ϕ_k is the geopotential height , $\overline{T_k}$ is the reference (global mean) temperature, and p_k is the pressure on the k-th level, and R_d is the dry gas constant.

$$B_{\zeta n} = \left\langle \Delta\zeta_n^m \overline{\Delta\zeta_n^m}^T \right\rangle, \quad B_{D_U n} = \left\langle \Delta D_{U n}^{\ m} \overline{\Delta D_{U n}^{\ m}}^T \right\rangle,$$

$$B_{\left(\begin{smallmatrix} T \\ P_s \end{smallmatrix}\right)_U n} = \left\langle \Delta\begin{pmatrix} T \\ p_S \end{pmatrix}_{U n}^{m} \overline{\Delta\begin{pmatrix} T \\ p_S \end{pmatrix}_{U n}^{m}}^T \right\rangle, \quad B_{q n} = \left\langle \Delta \ln q_n^m \overline{\Delta \ln q_n^m}^T \right\rangle \tag{3.5.9}$$

The vertical correlations of specific humidity between model levels lower than 30 (about 55hPa) and those higher than 31 are neglected to avoid spurious stratospheric humidity increments. Total variances of the control variables are rescaled by a factor of 0.81.

(d) Cholesky decomposition of background error covariance matrix

The background error covariance matrix mentioned above is decomposed by the Cholesky decomposition. It gives the independent and normalized (i.e. preconditioned) control variables Δy_n^m as follows:

$$\begin{aligned} J_n^{(x)} &= \sum_{m=-n}^{n} \frac{1}{2}\left(\Delta\overline{x}_n^m\right)^T B_n^{-1}\Delta x_n^m = \sum_{m=-n}^{n} \frac{1}{2}\left(\Delta\overline{x}_n^m\right)^T (L_n L_n^T)^{-1}\Delta x_n^m = \sum_{m=-n}^{n} \frac{1}{2}\left(\Delta\overline{y}_n^m\right)^T \Delta y_n^m \\ \Delta y_m^n &\equiv L_n^{-1}\Delta x_n^m \end{aligned} \tag{3.5.10}$$

where $J_n^{(x)}$ is a background error term for a control variable x at total wavenumber n, B_n is a background covariance matrix for x, L_n is a lower triangular matrix.

In summary, normalized control variables $\Delta y_n^m(k)$ are independent both horizontally and vertically, and the independent variables are normalized by the background error variance. The background term of the cost function is simplified as a summation of square of the normalized control variables at each wave number and each vertical level.

(e) Conversions from preconditioned control variables to analysis variables

The conversions from the preconditioned control variables to the analysis variables are performed by the following procedures:

$$\Delta y \xrightarrow{XCT2QD} \begin{pmatrix} \Delta\zeta \\ \Delta D_U \\ \Delta(ps,T)_U \\ \Delta \ln q \end{pmatrix} \xrightarrow{RO2BMA} \begin{pmatrix} \Delta\zeta \\ \Delta D_U \\ \Delta(ps,T)_U \\ \Delta \ln q \\ \Delta\phi_B \end{pmatrix} \xrightarrow[LNQ2Q]{\substack{UPT2PT \\ UDI2DI}} \begin{pmatrix} \Delta\zeta \\ \Delta D \\ \Delta(ps,T) \\ \Delta q \end{pmatrix} \tag{3.5.11}$$

First, the control variables (relative vorticity, unbalanced divergence, unbalanced temperature and surface pressure, the logarithm of specific humidity) on each model level in the spectral space are reconstructed by using Eq. (3.5.10) (XCT2QD in above figure). Then, the modified balance mass variable $\Delta\phi_B$ is calculated from the relative vorticity $\Delta\zeta$ by (3.5.7). The temperature and the surface pressure and the divergence are calculated by Eq. (3.5.4) from the unbalanced variables (UPT2PT and UDI2DI). The logarithm of specific humidity is converted to specific humidity (LNQ2Q). If the specific humidity of the first guess in the grid space is negative due to the wave-to-grid transform, Δq is set to be zero.

3.5.7 Observation terms

(a) Observational data

Observational data and departures (observation minus first guess) are given with the location through pre-analysis procedure. Reported surface pressure data at the station height and sea surface pressure data of surface observation are assimilated after converted onto the model surface height prior to assimilation. Satellite radiance data from ATOVS and microwave imagers are directly assimilated.

(b) Observation error correlations

The observation errors are estimated by innovation statistics. The vertical correlations of radio sonde data are not considered. Dense observations (satellite data or aviation data) are thinned in the pre-analysis procedure to avoid the horizontal error correlation.

(c) Conversion from analysis variables to observation variables

In 4D-Var, an observation at an observation position is simulated from the analysis variables at the surrounding gaussian grids by using variable conversions and spatial interpolation or extrapolation to the observation position.

Linear vertical interpolation is performed first with respect to logarithm of pressure. Extrapolation up to 0.36 hPa and under the model ground surface is also performed. Then linear horizontal interpolation and extrapolation to the North and South poles are carried out.

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