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Metrological traceability
and
Uncertainty of measurement

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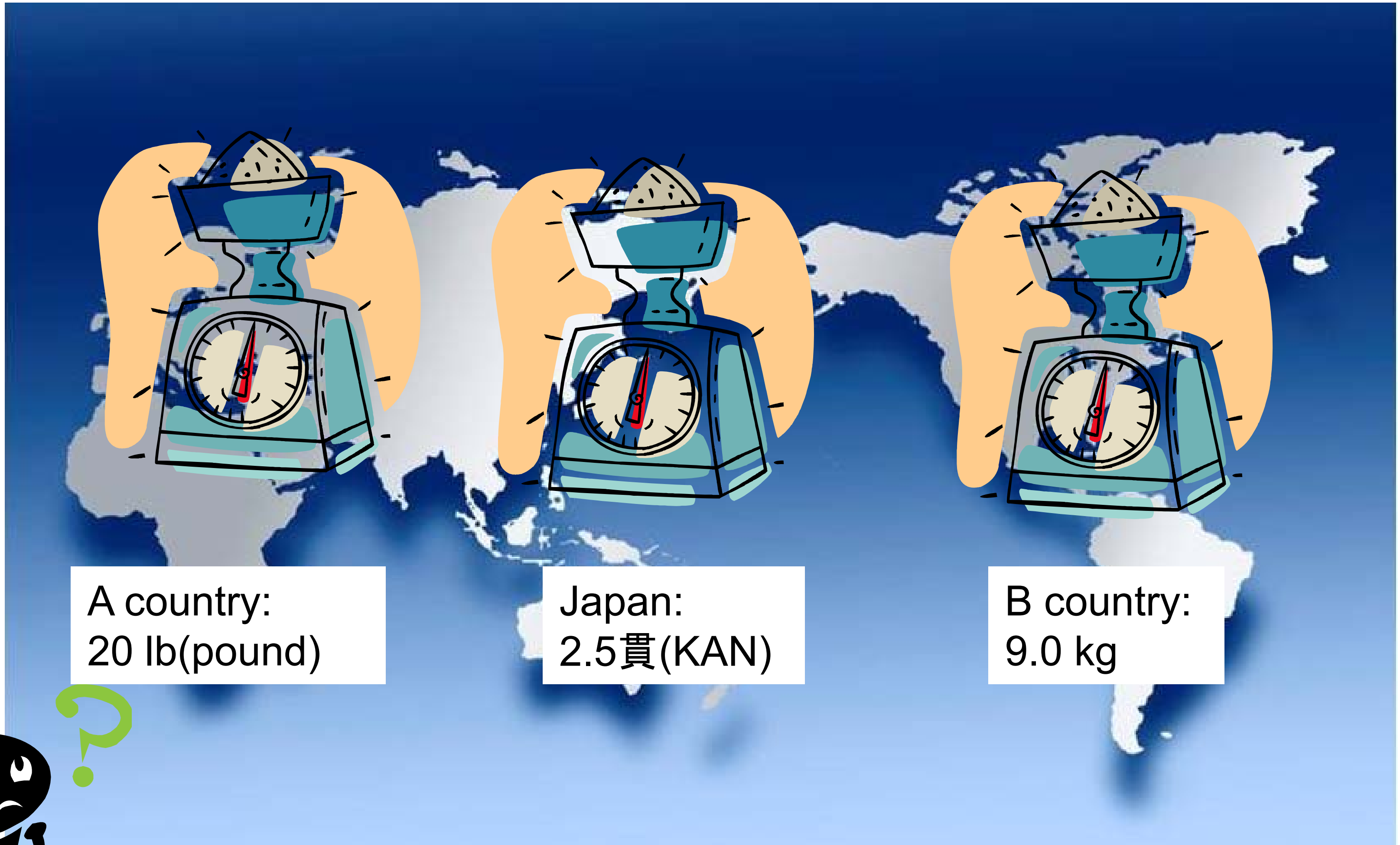
Metrological traceability

Why is “the SI” important?



Which is the heaviest?
How to convert units?

To unify the one unit in
the world.

The SI

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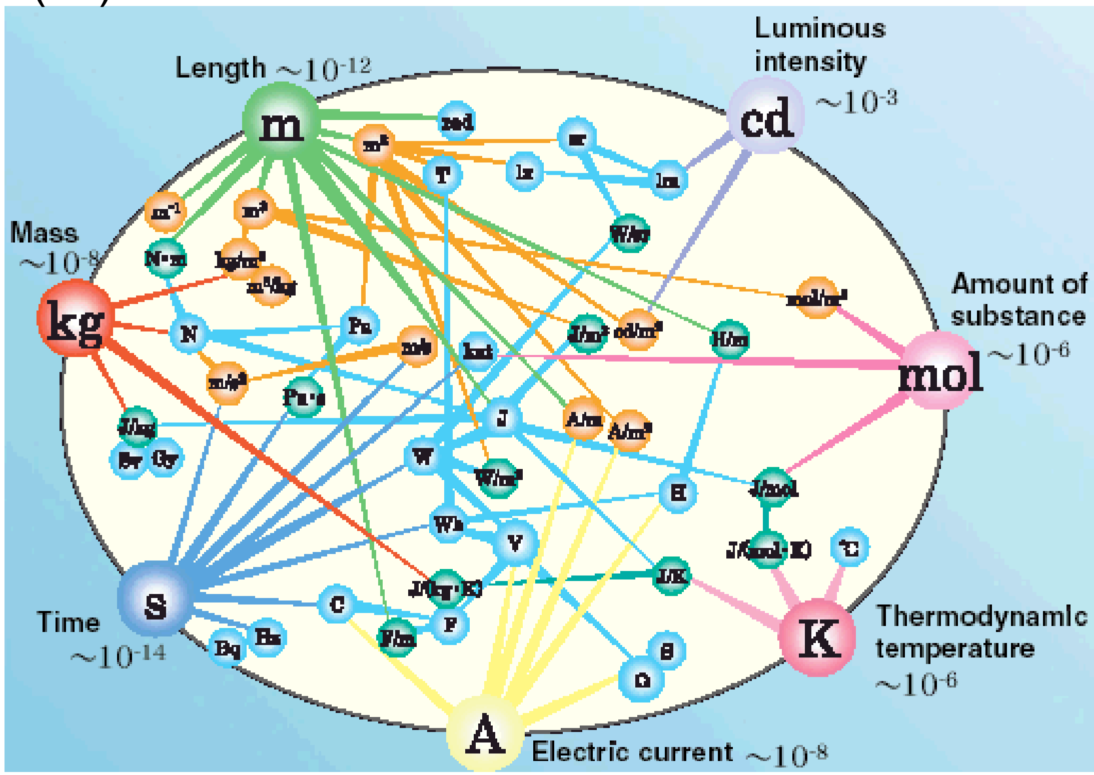
What is “the SI” ?

The International System of Units (SI)

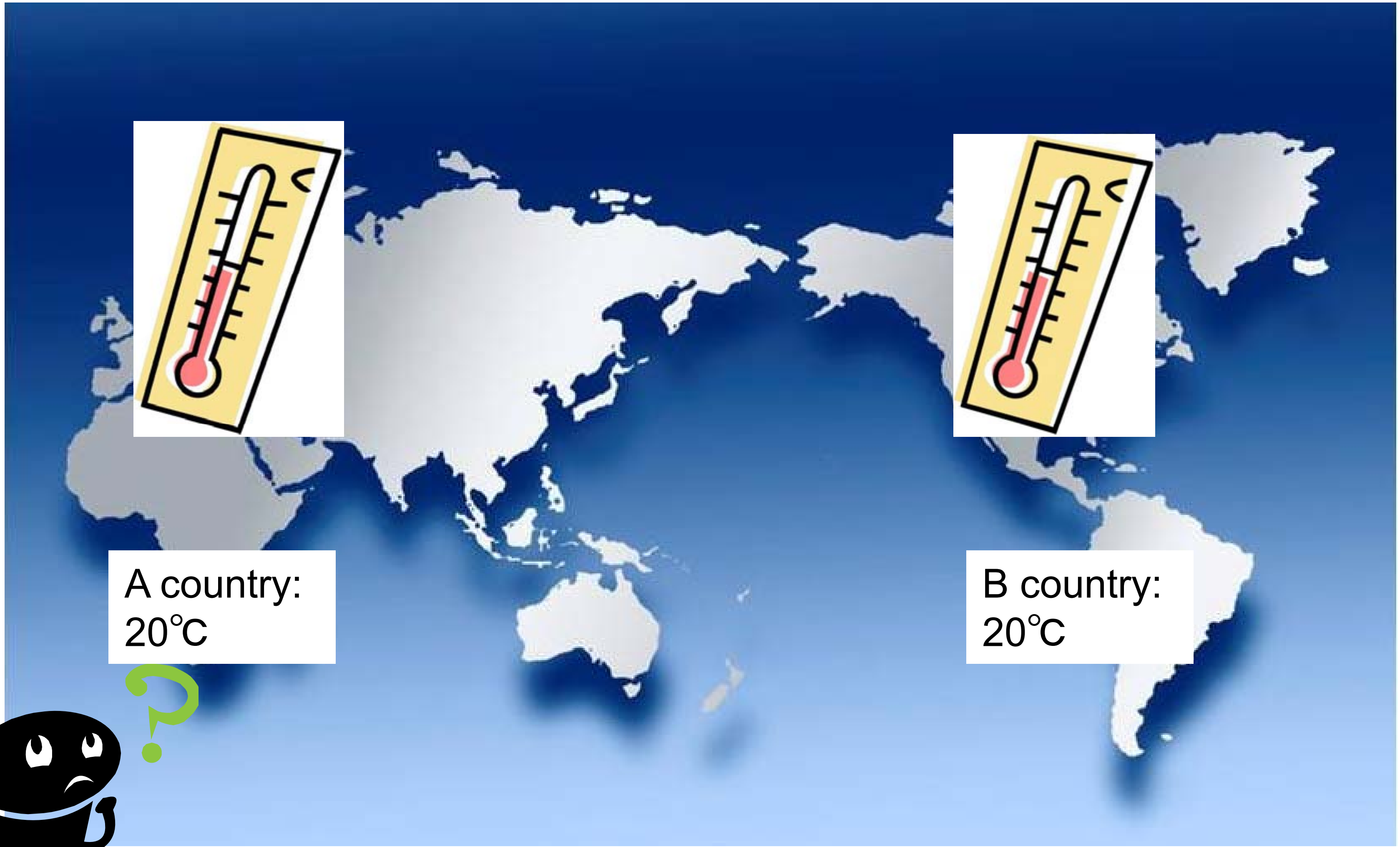
Based on the Metre Convention, the International System of Units (SI) is structured based on the seven base units: the meter (m), the kilogram (kg), the second (s), the ampere (A), the kelvin (K), the mole (mol) and the candela (cd).



The number on the right shoulder of each base unit indicates its degree of uncertainty (k = 2) .

http://www.nmij.jp/english/library/units/

Why is “traceability” important?



How is it proved that the measured temperature 20℃ of A country and 20℃ of B country as the same temperature?

What is “traceability” ?

“metrological traceability”

property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty

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What is “traceability” ?

The ILAC (International Laboratory Accreditation Cooperation) considers the elements for confirming metrological traceability to be an unbroken metrological traceability chain to an international measurement standard or a national measurement standard, a documented measurement uncertainty, a documented measurement procedure, accredited technical competence, metrological traceability to the SI, and calibration intervals.

The abbreviated term “traceability” is sometimes used to mean ‘metrological traceability’ as well as other concepts, such as ‘sample traceability’ or ‘document traceability’ or ‘instrument traceability’ or ‘material traceability’, where the history (“trace”) of an item is meant. Therefore, the full term of “metrological traceability” is preferred if there is any risk of confusion.

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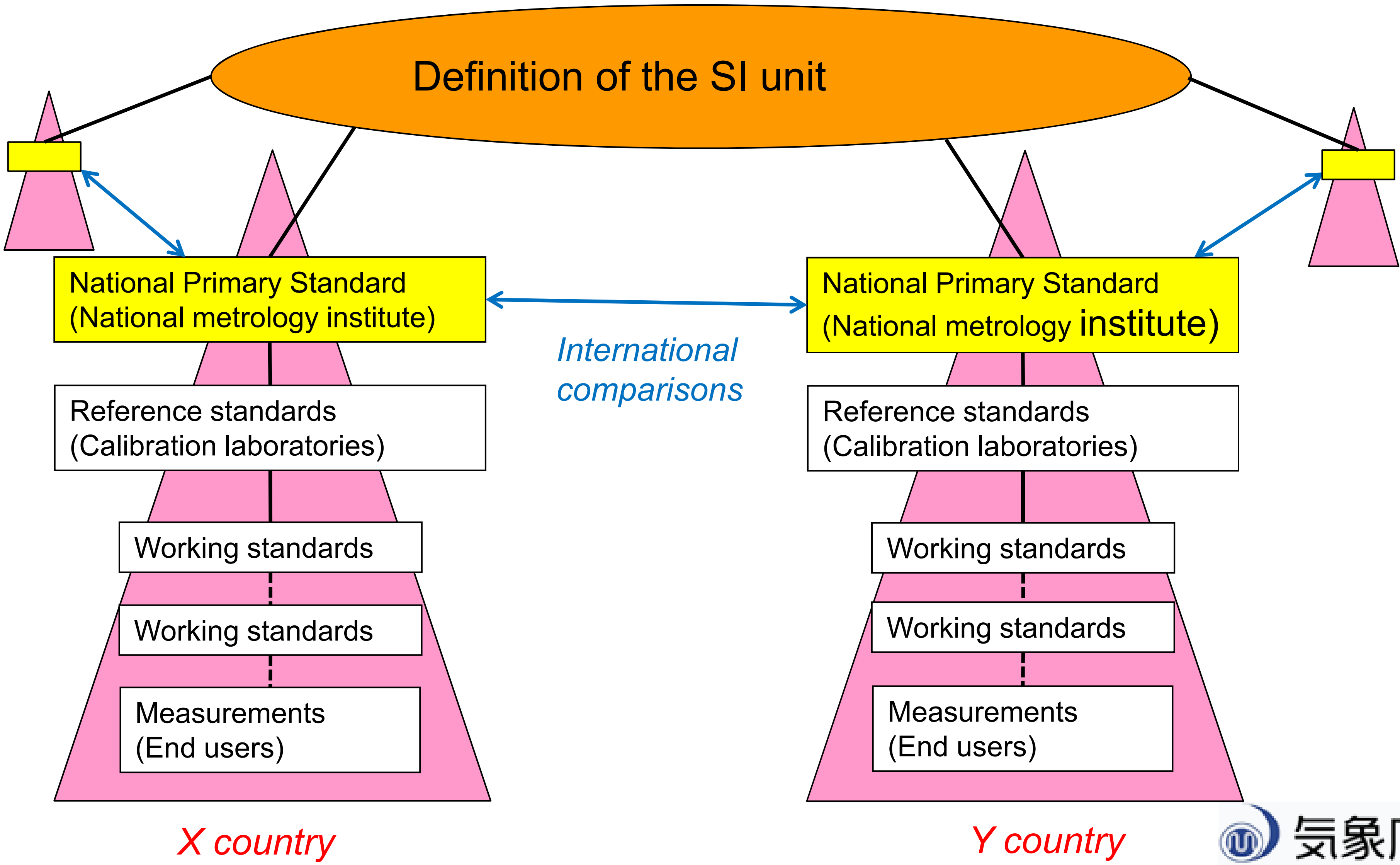
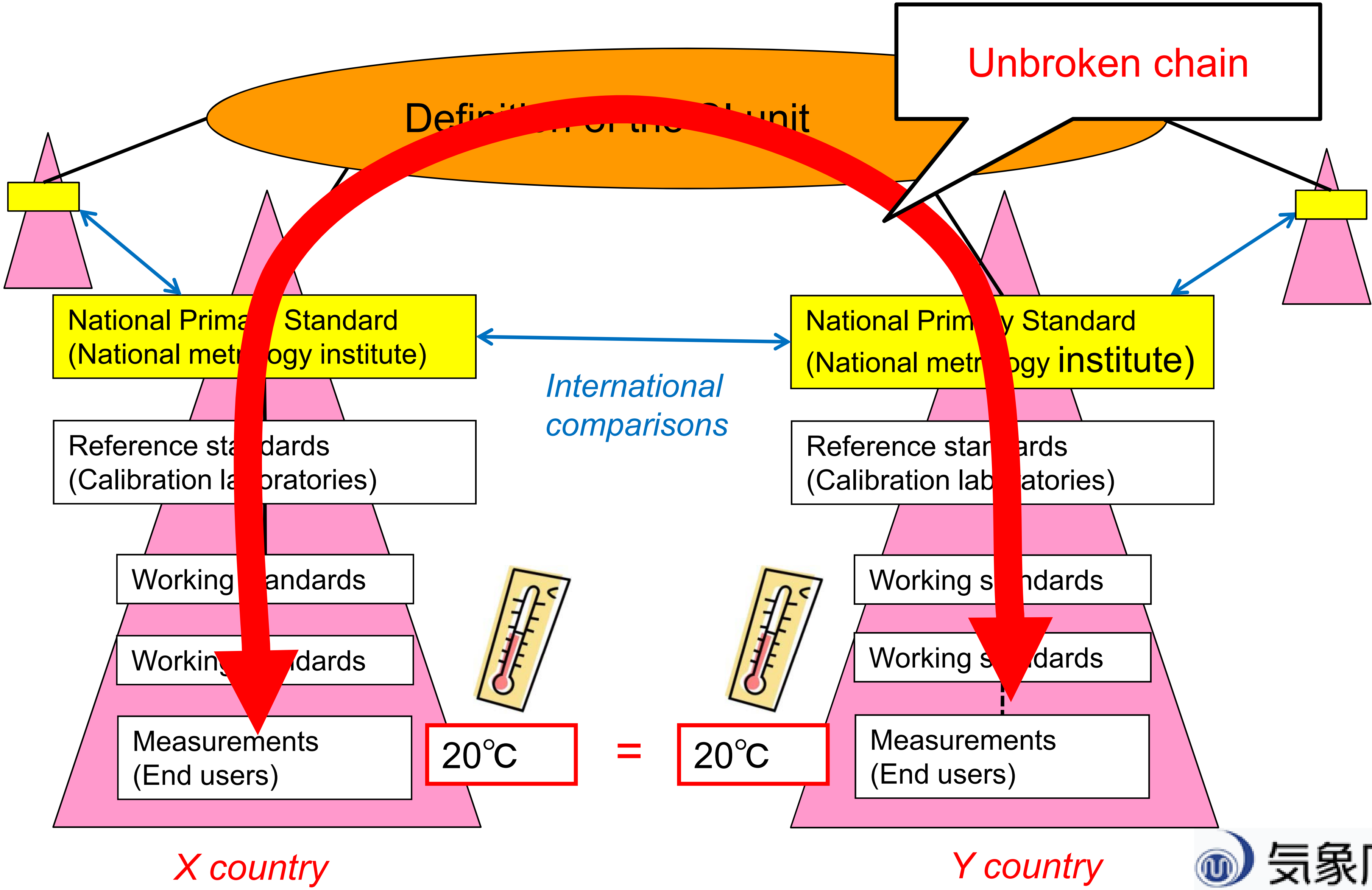
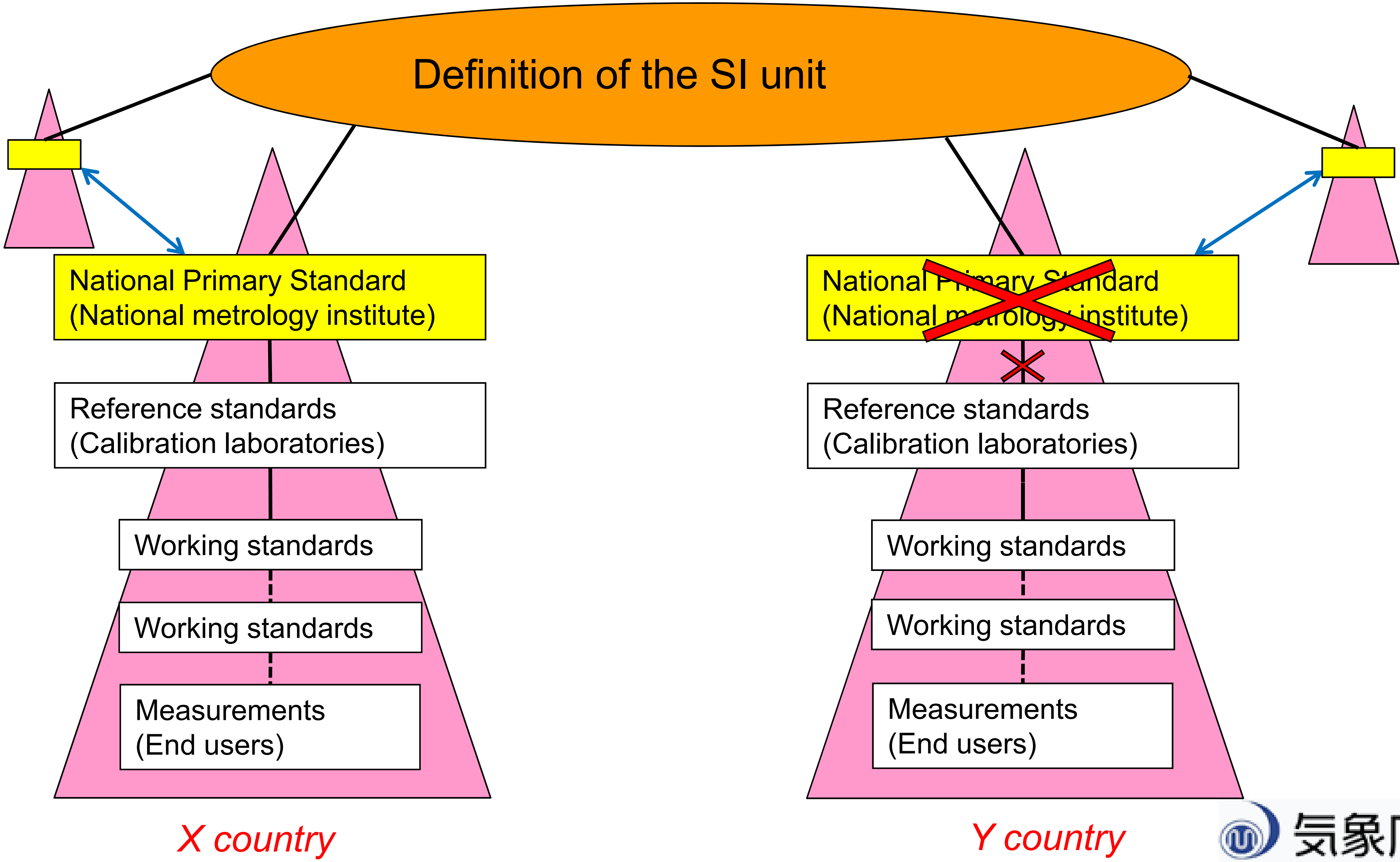
Traceability to the SI



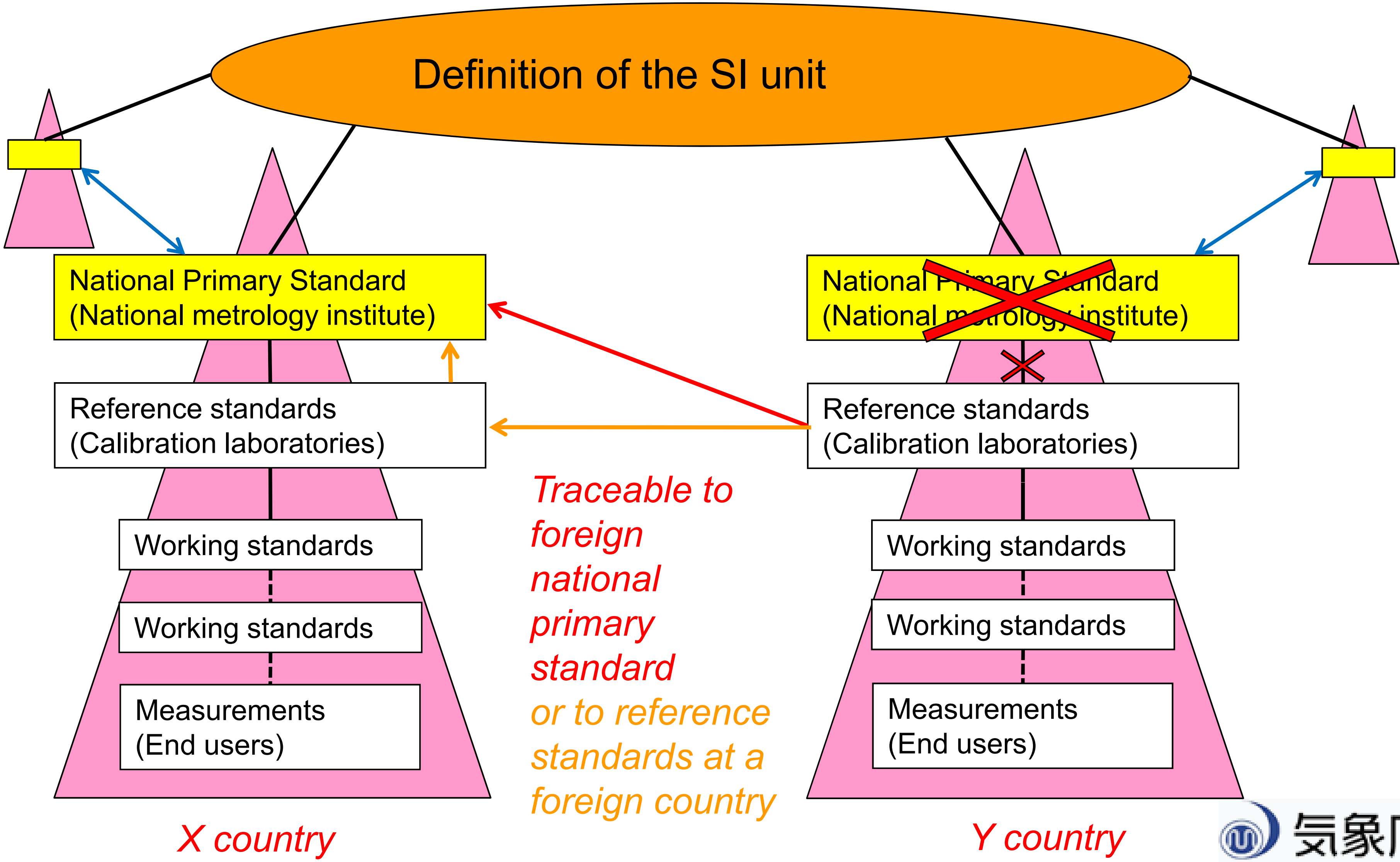
Traceability to the SI



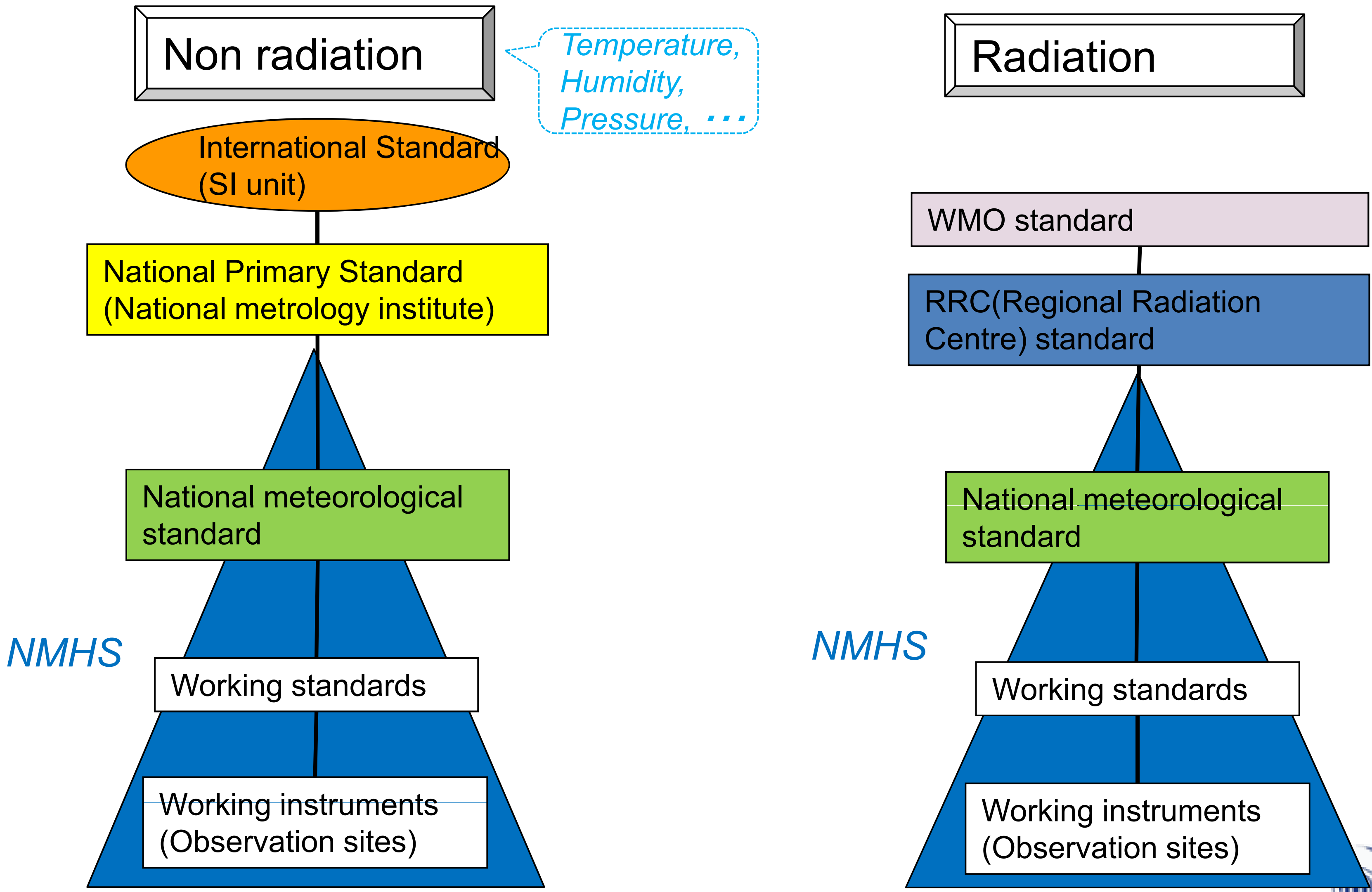
Traceability to the SI



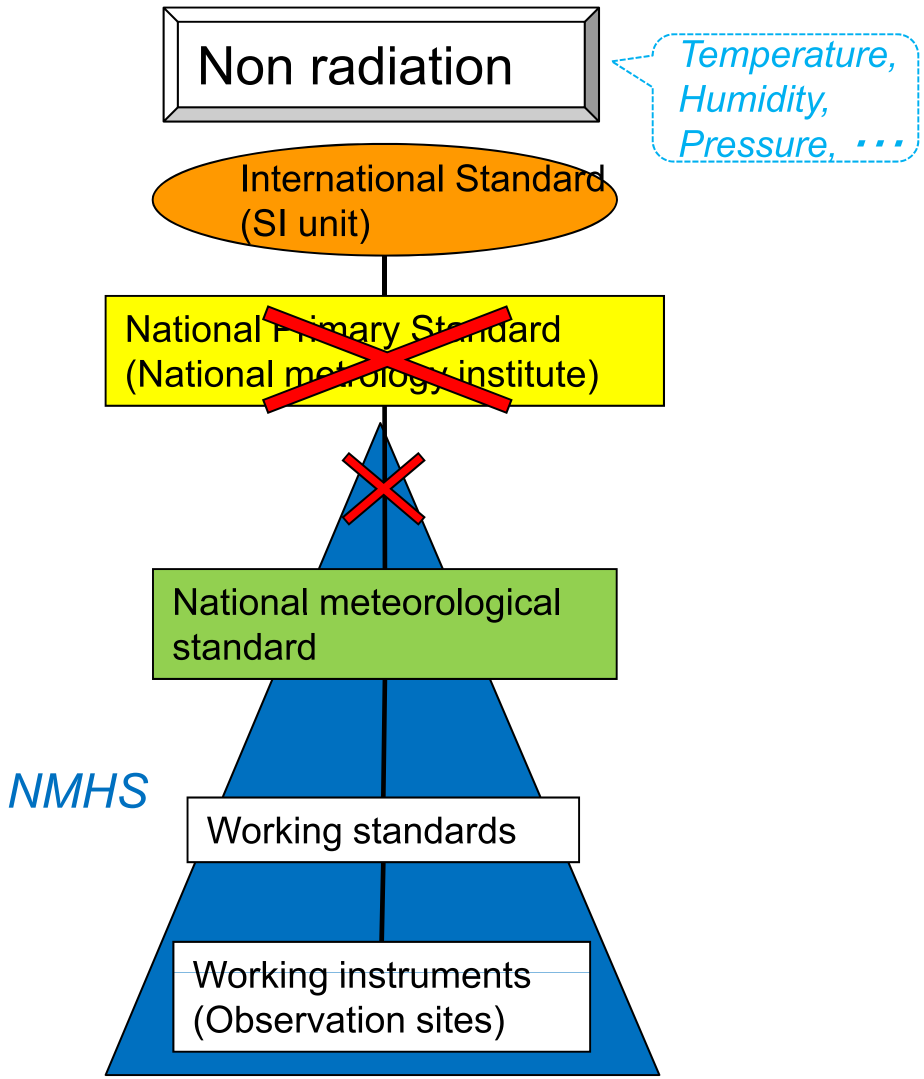
Traceability to the SI



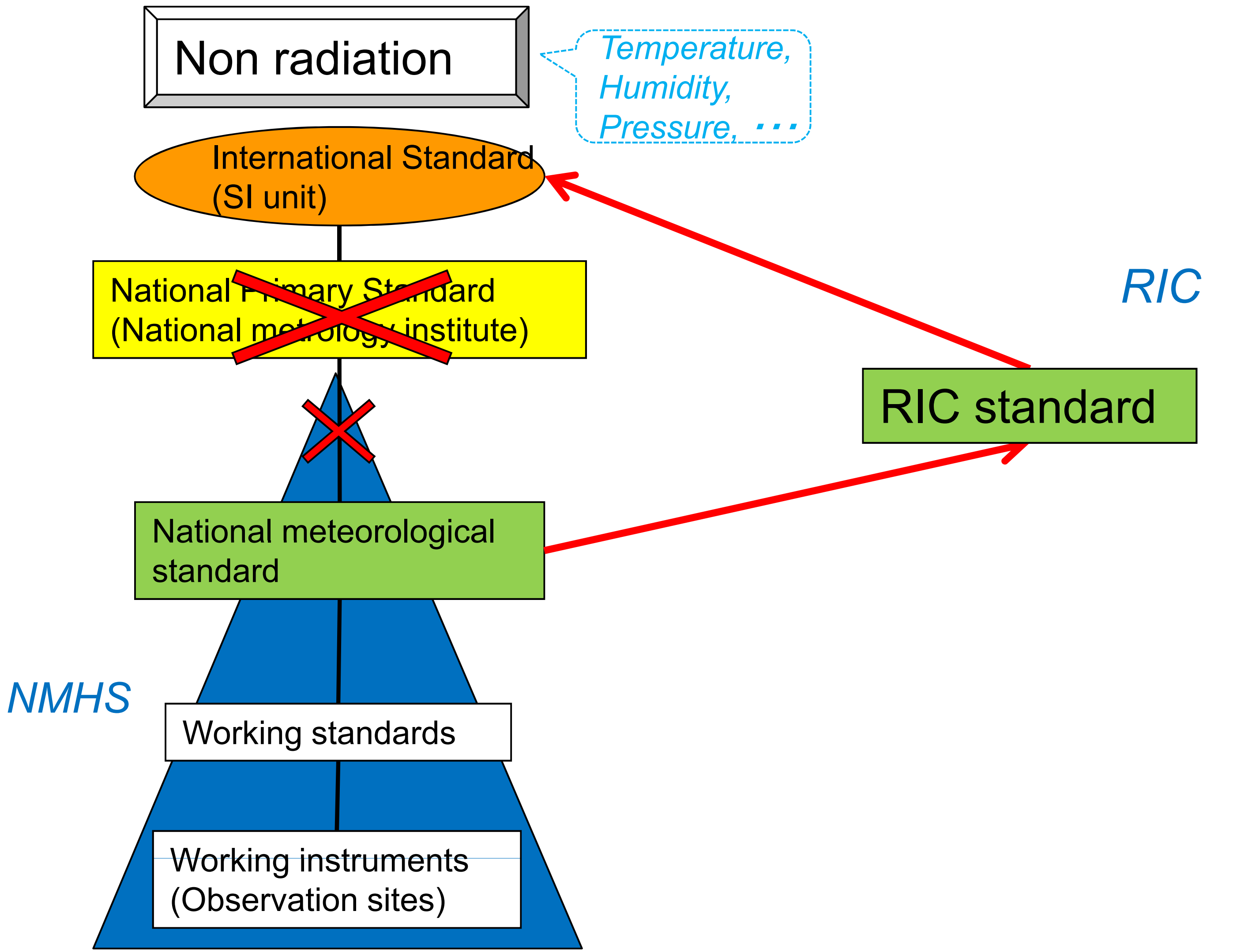
Traceability in meteorological instruments



Traceability in meteorological instruments



Traceability in meteorological instruments



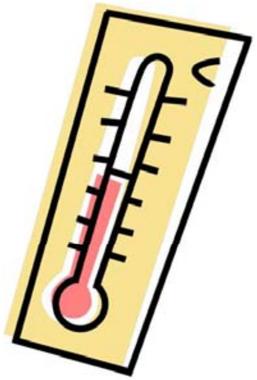
Uncertainty of measurement

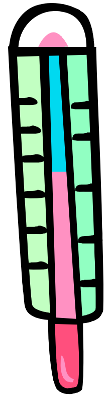
Outline

1. Introduction to measurement uncertainty
 (1) Background
 (2) Definition
2. Evaluation procedure of uncertainty
3. Example of uncertainty evaluation
 (Pressure measurement using by piston gauge)

1. Introduction to measurement uncertainty

Why is “measurement uncertainty” important?





Thermometer A:
20.0℃

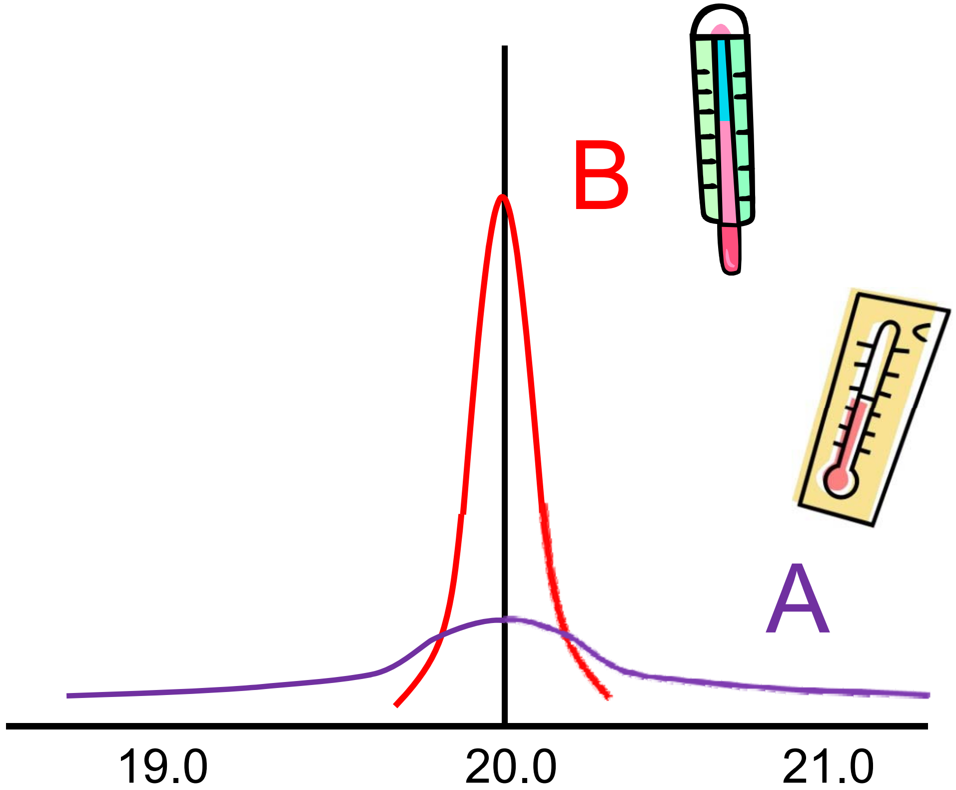
5 times measurements;
19.0℃,20.5℃,19.7℃,
21.0℃,19.8℃

Thermometer B :
20.0℃

5 times measurements;
19.9℃,20.0℃,20.1℃,
20.0℃,20.0℃

The average are same; 20.0℃.
But, are thermometers A and B the same performance?
How to evaluate the quality of thermometers?

Why is "measurement uncertainty" important?



Not only average values, but also information on dispersion are necessary.

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(Measurement) uncertainty

Why is "measurement uncertainty" important?

An unified index about quality of measurand:

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Measurement uncertainty;
non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used

Merits of using measurement uncertainty
(1) Easy to compare the measurement results in the world
(2) Proof of reliability of measurement results in the world

Measurand; quantity intended to be measured <VIM>

How to treat dispersion of the quantity values?

Previously

Error approach

(sometimes called “Traditional approach”

or “True value approach”)

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At present

Uncertainty approach

Why changed from “Error approach” to “Uncertainty approach”?

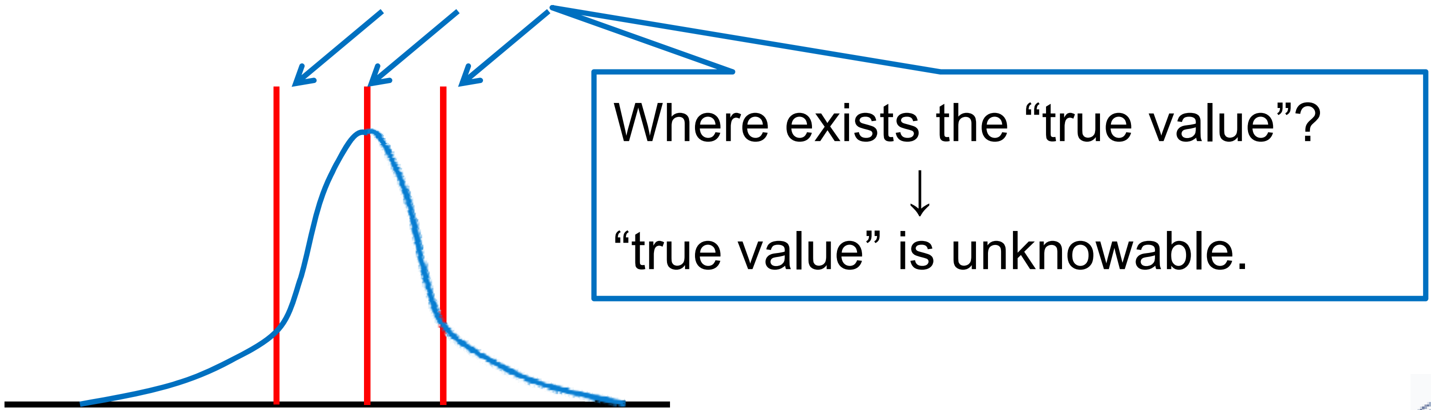
Background

Problems on error approach

(1) Traditional error estimation defines;

"Error = measured value – true value"

But, "true value" is unknowable in practice.



Background

Problems on error approach

(2)The definition of words which associated accuracy of measurement (for example ”accuracy”, “trueness”, “precision”, etc.) differ in regions , countries and specialized fields (physics, chemistry, engineering, etc.)

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Difficulty in discussion about the dispersion of the quantity values

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Background

To unify expression about reliability of measurement result was necessary.

- International review had started since around 1977 due to proposal by CIPM (International Committee of Weights and Measures).
- International document called "GUM" was published in 1993.

What is “GUM”?

Guide to the Expression of Uncertainty in Measurement

GUM→BIPM website
http://www.bipm.org/en/publications/guides/gum.html

What is “GUM”?

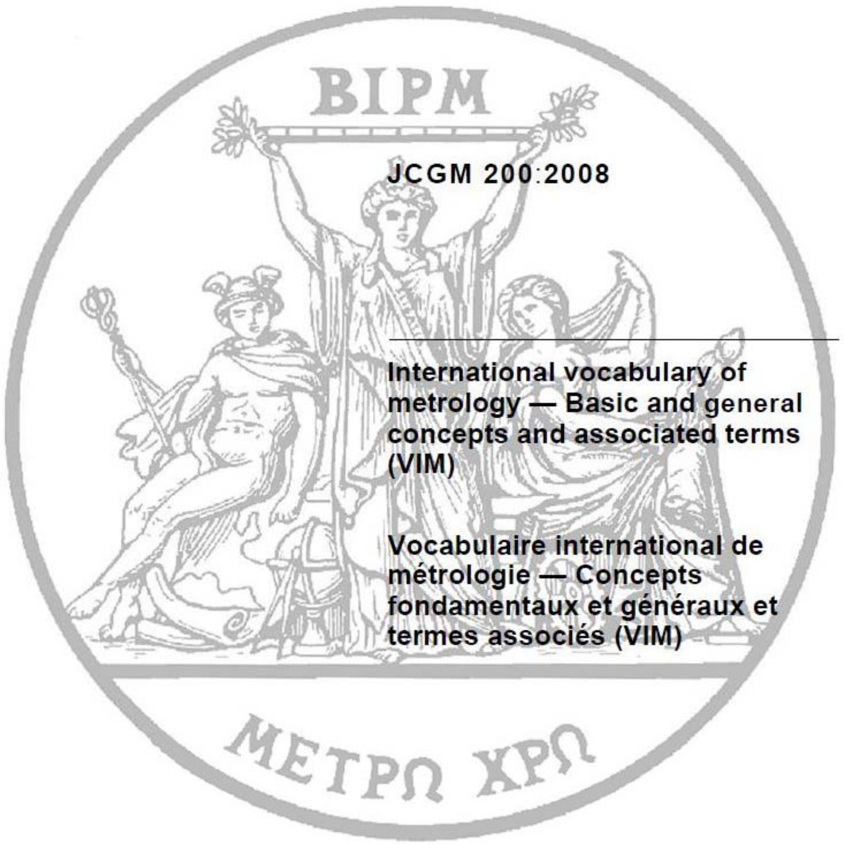
GUM(1st version) was published as collaborative editing by the following 7 organizations

BIPM(Bureau International des Poids et Mesures)
IEC(International Electrotechnical Commission)
IFCC(International Federation of Clinical Chemistry)
ISO(International Organization for Standardization)
IUPAC(International Union of Pure and Applied Chemistry)
IUPAP(International Union of Pure and Applied Physics)
OIML(International Organization of Legal Metrology)

In 2005, ILAC(International Laboratory Accreditation Cooperation) joined.
Latest version of GUM is published in 2008.

“VIM”

International Vocabulary of Metrology – Basic and General Concepts and Associated Terms



2. Evaluation procedure of uncertainty

How to evaluate uncertainty?

Evaluation procedure of uncertainty

Step	
Step1. Clarify measurement procedure and definition of measurand	List up of input quantities Set mathematical model of measurement
Step2. Evaluation of standard uncertainty about each input quantities	Type A evaluation Type B evaluation
Step3. Calculation of combined uncertainty	Propagation rule of uncertainty Budget sheet
Step4. Reporting uncertainty	Combined standard uncertainty Expanded uncertainty

Evaluation procedure of uncertainty

Step	
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Step1：Clarify measurement procedure and definition of measurand

(1)List up of input quantities and possible sources which effect measurement result

<GUM 3.3.2>

In practice, there are many possible sources of uncertainty in a measurement, including:

a) incomplete definition of the measurand;

b) imperfect realization of the definition of the measurand;

c) nonrepresentative sampling — the sample measured may not represent the defined measurand;

d) inadequate knowledge of the effects of environmental conditions on the measurement or imperfect measurement of environmental conditions;

Step1：Clarify measurement procedure and definition of measurand

e) personal bias in reading analogue instruments;

f) finite instrument resolution or discrimination threshold;

g) inexact values of measurement standards and reference materials;

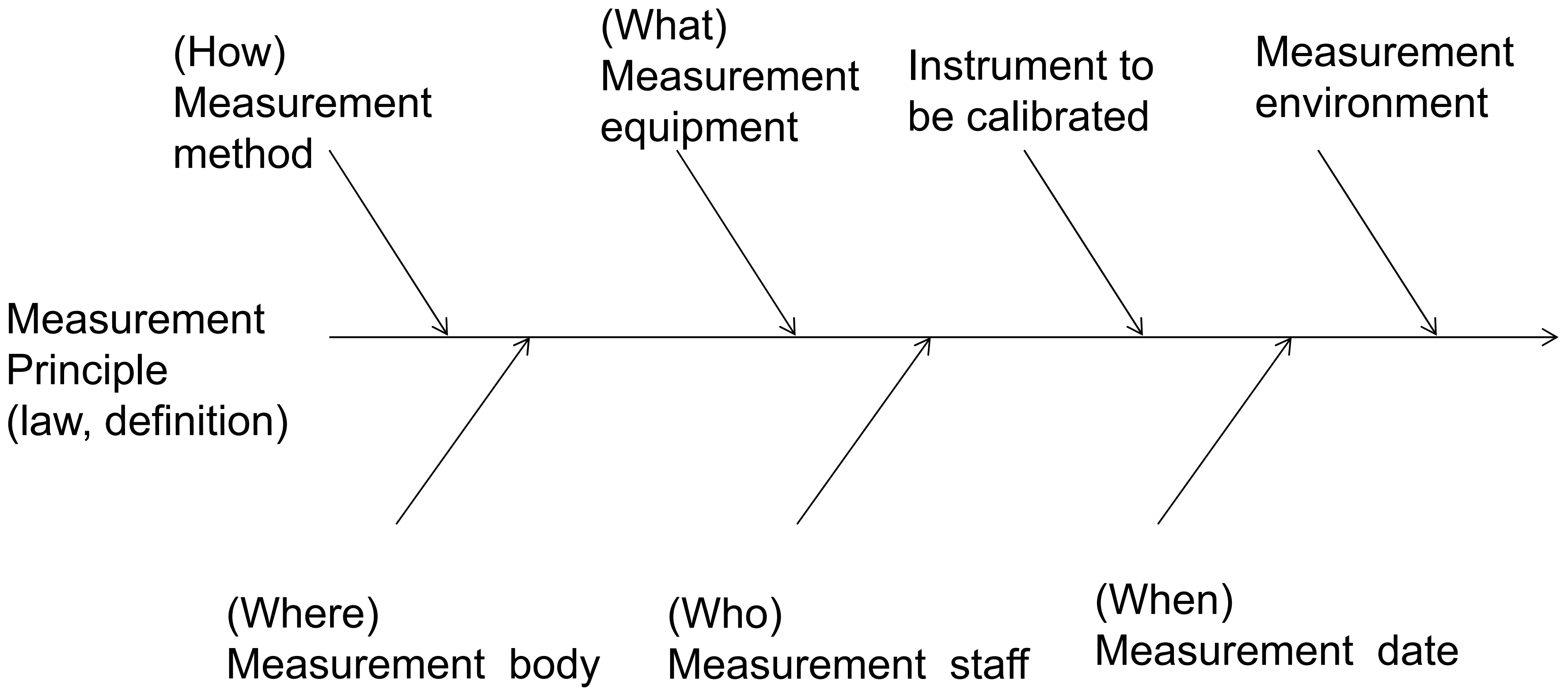
h) inexact values of constants and other parameters obtained from external sources and used in the data-reduction algorithm;

i) approximations and assumptions incorporated in the measurement method and procedure;

j) variations in repeated observations of the measurand under apparently identical conditions.

Step1：Clarify measurement procedure and definition of measurand

(1) List up of input quantities and possible sources which effect measurement results



Fishbone diagram

Step1：Clarify measurement procedure and definition of measurand

(2) Set mathematical model of measurement

$$y = f\left(x_1, x_2, \ldots, x_n\right)$$

Measurand Input quantity

Example;

P=F/A

P: Pressure, ←Measurand

F: Force, ← Input quantity

A: Area ← Input quantity

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Step1：Clarify measurement procedure and definition of measurand

The important things for the beginners;

(1) It is NOT necessary to try to evaluate all sources which effect measurement result from the beginning.

(2) Try to pick up sources which significantly effect measurement result .

Evaluation procedure of uncertainty

Step1. Clarify measurement procedure and definition of measurand
- List up of input quantities
- Set mathematical model of measurement

↓

Step2. Evaluation of standard uncertainty about each input quantities
- Type A evaluation
- Type B evaluation

↓

Step3. Calculation of combined uncertainty
- Propagation rule of uncertainty
- Budget sheet

↓

Step4. Reporting uncertainty
- Combined standard uncertainty
- Expanded uncertainty

How to evaluate standard uncertainty about each input quantities?

Two types evaluation methods are considered in GUM.

1. Type A evaluation (of measurement)
2. Type B evaluation (of measurement)

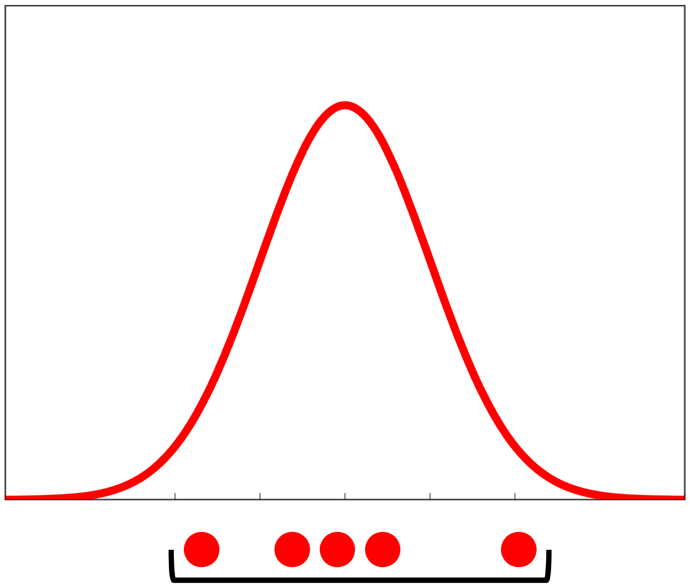
What is Type A and Type B evaluation (of measurement)?

1. Type A evaluation (of measurement)
   Evaluation of a component of measurement uncertainty by statistical analysis (measured values obtained from repeated observations).

2. Type B evaluation (of measurement)
   Evaluation of a component of measurement uncertainty determined by means other than Type A
   (previous data, certificate, handbooks, etc.).

→Bose type A and type B can be characterized by standard deviation and treated as same component when they are combined in the next step.

Type A evaluation (of measurement)



Measured values

Measured values from n independent observations;

$$X = \left[x_1, x_2, \ldots, x_n\right]$$

(1) Average;

$$\overline{X} = \frac{(x_1 + x_2 + ... + x_n)}{n}$$

Type A evaluation (of measurement)

(2) Experimental standard deviation; s

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n - 1}}$$

(3) Experimental standard deviation of the mean; $u(\overline{x})$

$$u(\overline{x}) = \frac{s}{\sqrt{n}}$$

→Standard uncertainty

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Type B evaluation (of measurement)

Evaluation of a component of measurement uncertainty determined by means other than Type A

Type B evaluations are founded on a priori distributions.

– previous measurement data;

– experience with or general knowledge of the behaviour and properties of relevant materials and instruments;

– manufacturer's specifications;

– data provided in calibration and other certificates;

– uncertainties assigned to reference data taken from handbooks.

Type B evaluation (of measurement)

Type B evaluation of measurement uncertainty, can also be characterized by standard deviations, evaluated from probability density functions based on experience or other information

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Input quantities are suppose to some distribution patterns;

normal distribution,

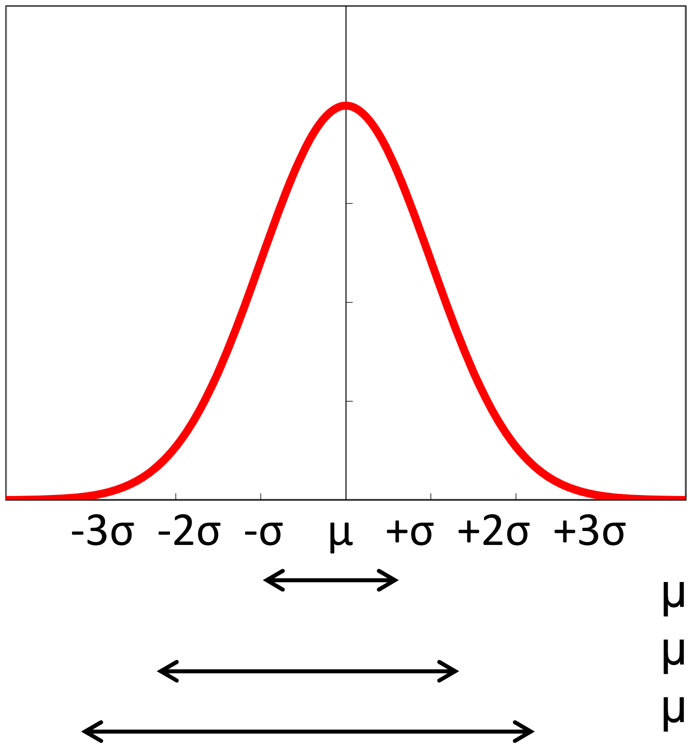
rectangular distribution,

triangle distribution…

Type B evaluation (of measurement)

If it is assumed that the input quantity Xi is a normal distribution with standard deviation σ,

Normal distribution



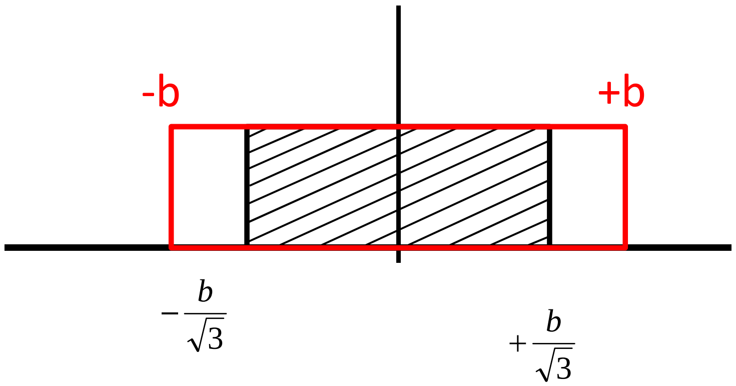
Standard uncertainty of Xi; $u(X_i)$

$$u(X_i) = \sigma$$

μ±1σ: 68.3%
μ±2σ: 95.5%
μ±3σ: 99.7%

Type B evaluation (of measurement)

rectangular distribution



If it is assumed that little information is available about the input quantity Xi and that all one can do is suppose that Xi is described by a symmetric, rectangular a priori probability distribution of lower bound –b, upper bound +b

Standard uncertainty of Xi; $u(X_i)$

$$u(X_i) = \frac{b}{\sqrt{3}}$$

Evaluation procedure of uncertainty

Step1. Clarify measurement procedure and definition of measurand

- List up of input quantities
- Set mathematical model of measurement

Step2. Evaluation of standard uncertainty about each input quantities

- Type A evaluation
- Type B evaluation

Step3. Calculation of combined uncertainty

- Propagation rule of uncertainty
- Budget sheet

Step4. Reporting uncertainty

- Combined standard uncertainty
- Expanded uncertainty

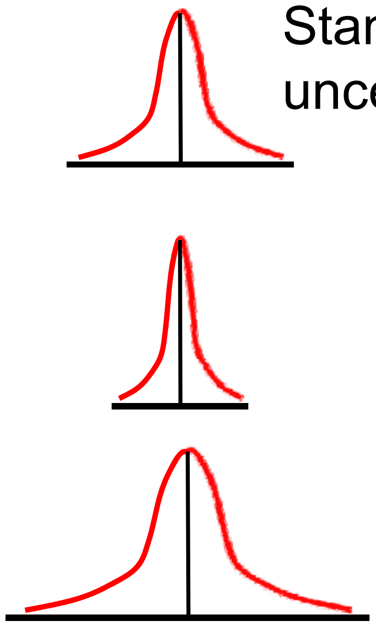
Step3．Calculation of combined uncertainty

How to combine uncertainties?

Square and add
No distinction type A and type B

This is the rule of GUM to combine uncertainties.

Input quantity ; x_1

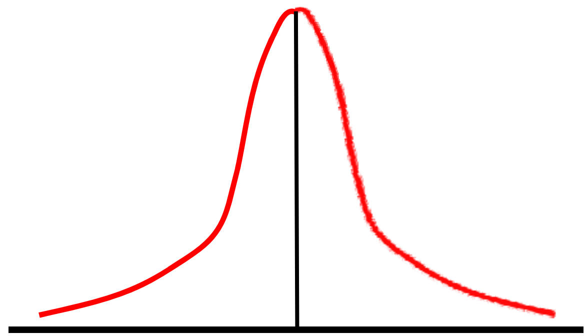


Standard uncertainty ; σ_1

x_2 σ_2

x_3 σ_3

⋮ ⋮



Propagation rule of uncertainty

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \cdots$$

Step3. Calculation of combined uncertainty

$$y = f(x_1, x_2, \ldots, x_n)$$

In the case of no correlation among input quantities $(x_1, x_2, \ldots, x_n)$,

combined uncertainty ; $u_c(y)$

$$u_c^2(y) = [\underline{c_1}\,\underline{u(x_1)}]^2 + [c_2 u(x_2)]^2 + \ldots + [c_n u(x_n)]^2$$

Sensitivity coefficients

Standard uncertainty

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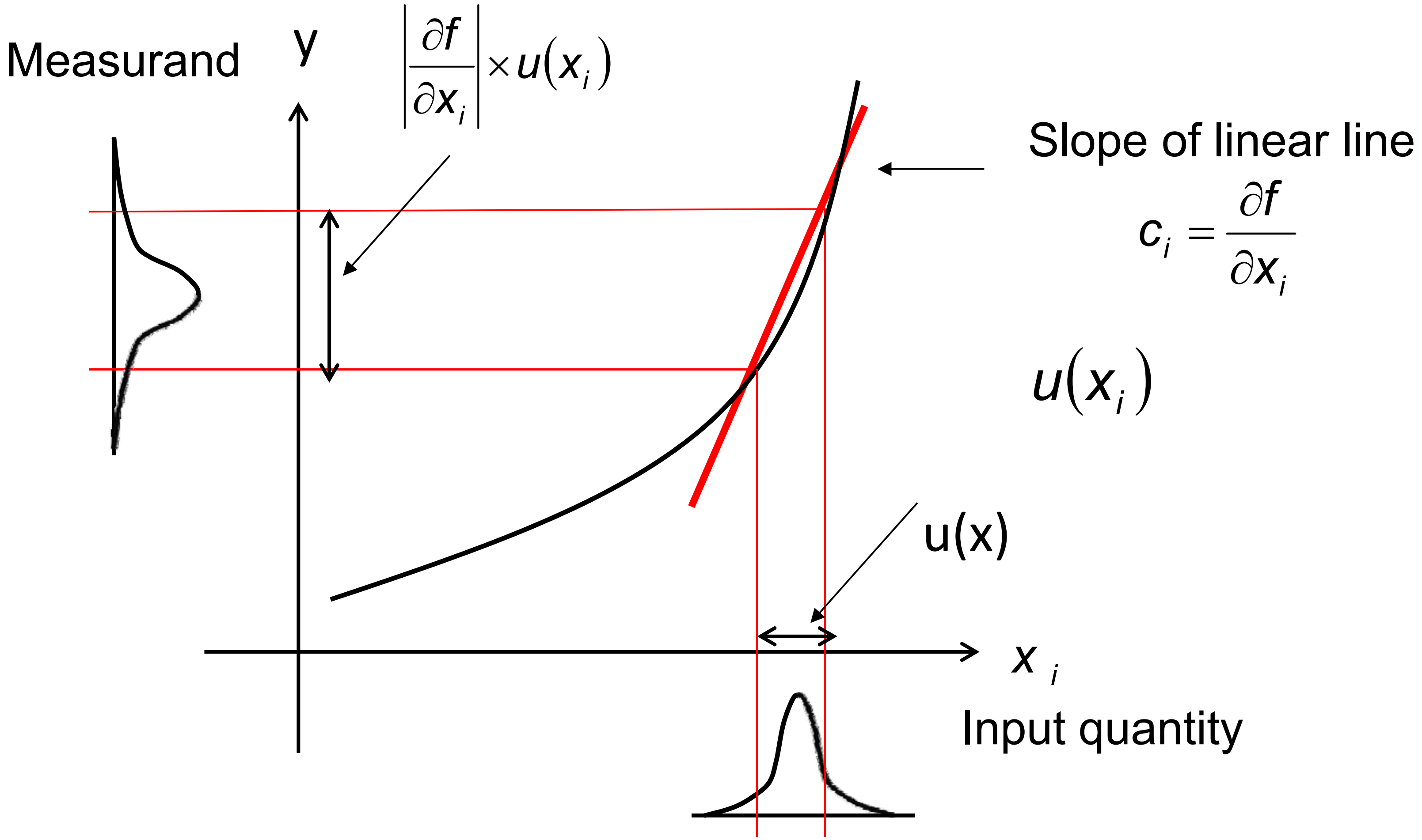
Sensitivity coefficients

Sensitivity coefficients;

As input data “x” changes, how measurand “y" changes

→partial differential coefficient



Budget sheet

Table H.1 — Summary of standard uncertainty components

Standard uncertainty component $u(x_i)$	Source of uncertainty	Value of standard uncertainty $u(x_i)$	$c_i \equiv \partial f / \partial x_i$	$u_i(l) \equiv \lvert c_i \rvert u(x_i)$ (nm)	Degrees of freedom
$u(l_S)$	Calibration of standard end gauge	25 nm	1	25	18
$u(d)$	Measured difference between end gauges	9,7 nm	1	9,7	25,6
$u(\bar{d})$	repeated observations	5,8 nm			24
$u(d_1)$	random effects of comparator	3,9 nm			5
$u(d_2)$	systematic effects of comparator	6,7 nm			8
$u(\alpha_S)$	Thermal expansion coefficient of standard end gauge	$1{,}2 \times 10^{-6}$ °C^{-1}	0	0	
$u(\theta)$	Temperature of test bed	0,41 °C	0	0	
$u(\bar{\theta})$	mean temperature of bed	0,2 °C			
$u(\Delta)$	cyclic variation of temperature of room	0,35 °C			
$u(\delta\alpha)$	Difference in expansion coefficients of end gauges	$0{,}58 \times 10^{-6}$ °C^{-1}	$-l_S\theta$	2,9	50
$u(\delta\theta)$	Difference in temperatures of end gauges	0,029 °C	$-l_S\alpha_S$	16,6	2

$$u_c^2(l) = \sum u_i^2(l) = 1\,002\ \text{nm}^2$$

$$u_c(l) = 32\ \text{nm}$$

$$\nu_{\text{eff}}(l) = 16$$

Evaluation procedure of uncertainty

Step1. Clarify measurement procedure and definition of measurand

- List up of input quantities
- Set mathematical model of measurement

↓

Step2. Evaluation of standard uncertainty about each input quantities

- Type A evaluation
- Type B evaluation

↓

Step3. Calculation of combined uncertainty

- Propagation rule of uncertainty
- Budget sheet

↓

Step4. Reporting uncertainty

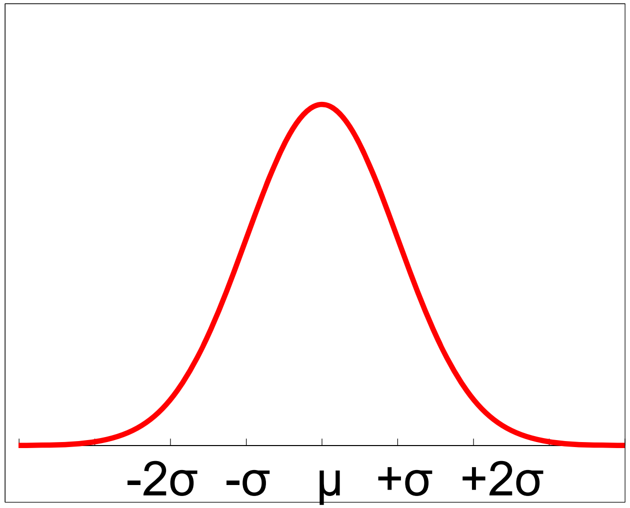
- Combined standard uncertainty
- Expanded uncertainty

Step4. Reporting uncertainty

(1) Combined standard uncertainty; $u_c(y)$

→Correspond to standard deviation(σ)

(2) Expanded uncertainty; $U = k \times u_c(y)$



k: coverage factor

$\pm U$: means confidence interval

k : usually used 2 - 3

k =2: About 95% of measurand is usually supposed to be in ±U

Step4. Reporting uncertainty

(1) Combined standard uncertainty; $u_c(y)$

<Example>
a nominally 100 g standard of mass m_s;
“m_s = 100,021 47 g with (a combined standard uncertainty)
u_c = 0,35 mg.”

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This means;
About 68 % of values exists 100,021 47±0,35 mg.

It usually suffices to quote u(y) and U
to at most two significant digits,

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Step4. Reporting uncertainty

(2) Expanded uncertainty; $U = k \times u_c(y)$

<Example>
"m_s = (100,021 47 ± 0,000 79) g, where the number following the symbol ± is the numerical value of (an expanded uncertainty) $U = ku_c$, with U determined from (a combined standard uncertainty) u_c = 0,35 mg and (a coverage factor) k = 2,26 based on the t-distribution for ν = 9 degrees of freedom, and defines an interval estimated to have a level of confidence of 95 percent."

Table G.2 — Value of $t_p(\nu)$ from the t-distribution for degrees of freedom ν that defines an interval $-t_p(\nu)$ to $+t_p(\nu)$ that encompasses the fraction p of the distribution

Degrees of freedom ν	Fraction p in percent					
	68,27 a)	90	95	95,45 a)	99	99,73 a)
1	1,84	6,31	12,71	13,97	63,66	235,80
2	1,32	2,92	4,30	4,53	9,92	19,21
3	1,20	2,35	3,18	3,31	5,84	9,22
4	1,14	2,13	2,78	2,87	4,60	6,62
5	1,11	2,02	2,57	2,65	4,03	5,51
6	1,09	1,94	2,45	2,52	3,71	4,90
7	1,08	1,89	2,36	2,43	3,50	4,53
8	1,07	1,86	2,31	2,37	3,36	4,28
9	1,06	1,83	2,26	2,32	3,25	4,09
10	1,05	1,81	2,23	2,28	3,17	3,96
11	1,05	1,80	2,20	2,25	3,11	3,85
12	1,04	1,78	2,18	2,23	3,05	3,76
13	1,04	1,77	2,16	2,21	3,01	3,69
14	1,04	1,76	2,14	2,20	2,98	3,64
15	1,03	1,75	2,13	2,18	2,95	3,59
16	1,03	1,75	2,12	2,17	2,92	3,54
17	1,03	1,74	2,11	2,16	2,90	3,51
18	1,03	1,73	2,10	2,15	2,88	3,48
19	1,03	1,73	2,09	2,14	2,86	3,45
20	1,03	1,72	2,09	2,13	2,85	3,42
25	1,02	1,71	2,06	2,11	2,79	3,33
30	1,02	1,70	2,04	2,09	2,75	3,27
35	1,01	1,70	2,03	2,07	2,72	3,23
40	1,01	1,68	2,02	2,06	2,70	3,20
45	1,01	1,68	2,01	2,06	2,69	3,18
50	1,01	1,68	2,01	2,05	2,68	3,16
100	1,005	1,660	1,984	2,025	2,626	3,077
∞	1,000	1,645	1,960	2,000	2,576	3,000

a) For a quantity z described by a normal distribution with expectation μ_z and standard deviation σ, the interval $\mu_z \pm k\sigma$ encompasses $p = 68{,}27$ percent, 95,45 percent and 99,73 percent of the distribution for $k = 1$, 2 and 3, respectively.

Degrees of freedom; ν

Level of confidence of approximately 95 %

ν	k
1	→ 12.71
5	→ 2.57
10	→ 2.23
∞	→ 1.96

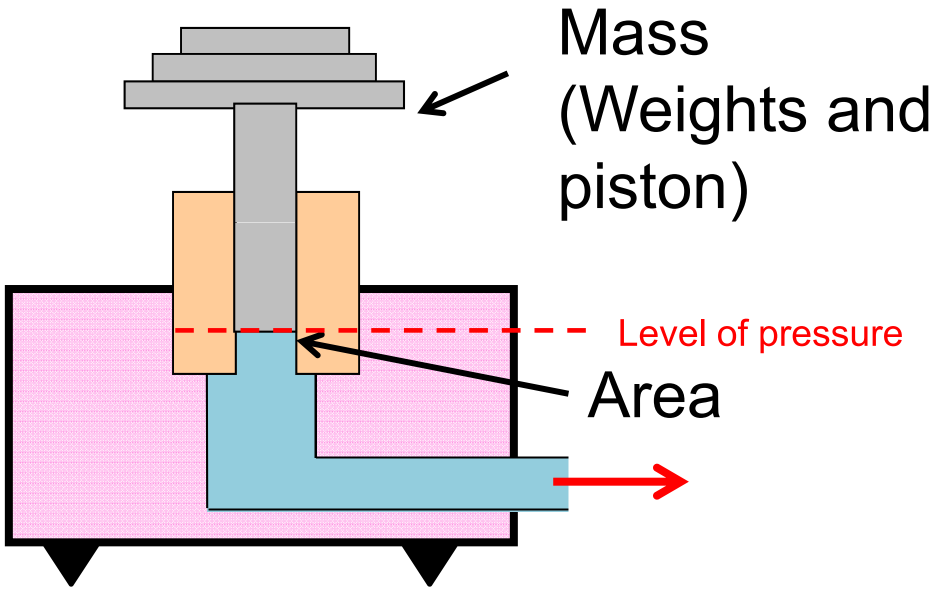
<GUM Table G.2>

3. Example of uncertainty evaluation

Pressure measurement using by piston gauge

Example situation

(a) Pressure generated by a piston gauge.
(b) Mass (Weights and Piston) are measured five times to calculate generated pressure by this gauge.
The measured values are" 1.03kg, 0.98kg, 0.99kg, 10.1kg, 0.99kg ".
(c) The area of the piston is $0.0001m^2 \pm 0.000001m^2$ ($1cm^2 \pm 0.1cm^2$) according to the manufacture's specification.
(d) The gravity of this calibration place is constant value ($10m/s^2$).



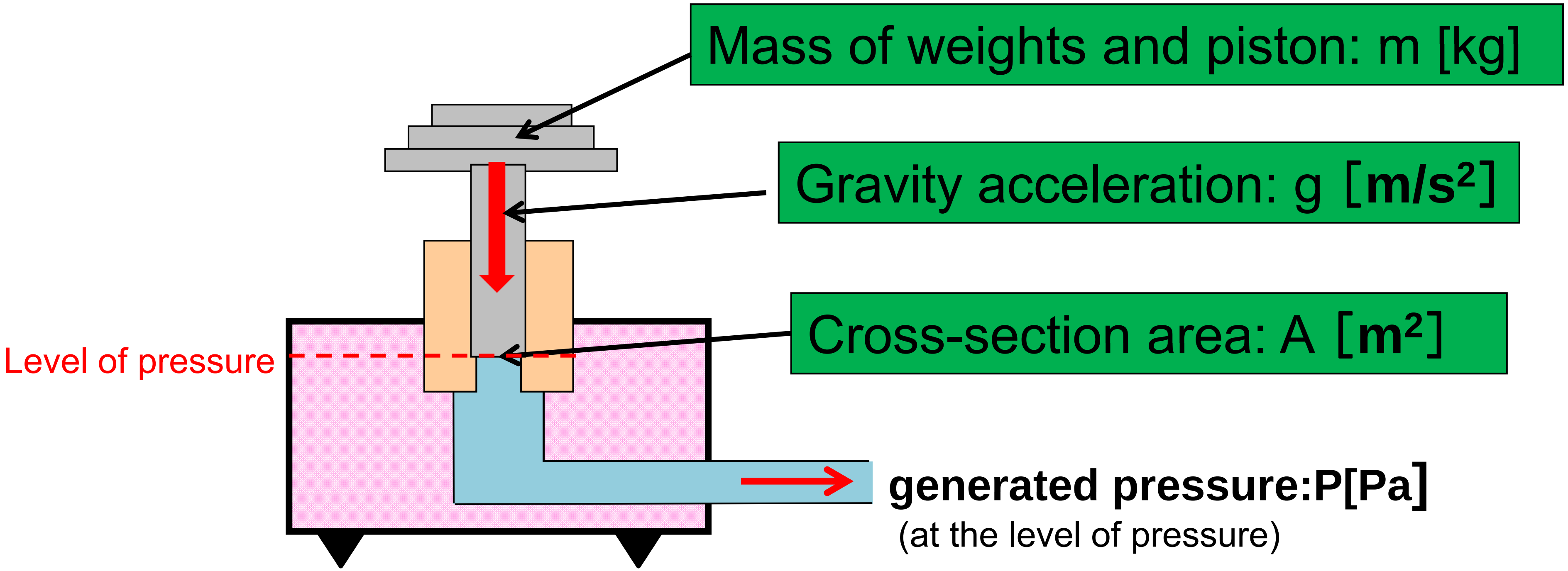
generated pressure: P
(at the level of pressure)

Question

(1) What is the average of generated pressure?

(2) What is the uncertainty of generated pressure?

List up of input quantities;



Set mathematical model of measurement;

$$P = \frac{mg}{A} \qquad [Pa] = \frac{[kg][m/s^2]}{[m^2]}$$

(1)What is the average of generated pressure?

$$\overline{m} = (1.03 + 0.98 + 0.99 + 1.01 + 0.99)$$

=1.00[kg]

g = 10.0[m/s^2]
A = 0.0001[m^2]

$$P = \frac{\overline{m}g}{A} = \frac{1.00 \times 10.0}{0.0001} = 100000\ [Pa]\,(= 1000\ [hPa])$$

Answer; 100000[Pa] (1000[hPa])

(2)What is the uncertainty of generated pressure?

$$P = \frac{mg}{A}$$

Evaluate uncertainty about each input quantities (m, g, A), respectively.

"g"

In this case, "g" is treated as constant number.

So, uncertainty of "g" is NOT considered.

Evaluate uncertainty about each input quantities (m, A), respectively.

(2)What is the uncertainty of generated pressure?

Budget sheet

Input quantities	**Standard uncertainty** $u(x_i)$	**Evaluation method**	**Sensitivity coefficients** c_i	**Contribution to** $u(y)$ $\lvert c_i \rvert u(x_i)$
Mass: m				
Area: A				

(2)What is the uncertainty of generated pressure?

Budget sheet

Input quantities	**Standard uncertainty** $u(x_i)$	**Evaluation method**	**Sensitivity coefficients** c_i	**Contribution to** $u(y)$ $\lvert c_i \rvert u(x_i)$
Mass: m	?	?		
Area: A				

(2)What is the uncertainty of generated pressure?

(a)m

"m" is measured values obtained from repeated observations

→Type A evaluation

Average;

$$\overline{m} = \frac{(m_1 + m_2 + ... + m_n)}{n} = 1.00[kg]$$

Experimental standard deviation;

$$s = \sqrt{\frac{\sum_{i=1}^{n}(m_i - \overline{m})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n}(m_i - \overline{m})^2}{5-1}} = 0.02\ [kg]$$

Experimental standard deviation of the mean;

$$u(\overline{m}) = \frac{s}{\sqrt{n}} = \frac{0.02}{\sqrt{5}} = 0.00894[kg]$$

Budget sheet

Input quantities	Standard uncertainty $u(x_i)$	Evaluation method	Sensitivity coefficients c_i	Contribution to $u(y)$ $\lvert c_i \rvert u(x_i)$
Mass: m	$u(\overline{m}) = 0.00894[kg]$	Type A (five times measurements)		
Area: A				

Budget sheet

Input quantities	**Standard uncertainty** $u(x_i)$	**Evaluation method**	**Sensitivity coefficients** c_i	**Contribution to** $u(y)$ $\lvert c_i \rvert u(x_i)$
Mass: m	$u(\overline{m}) = 0.00894[kg]$	Type A (five times measurements)	?	?
Area: A				

Sensitivity coefficients;

$$c_i = \frac{\partial f}{\partial x_i}$$

$$g = 10.0\left[m/s^2\right], A = 0.0001\left[m^2\right]$$

$$c_m = \frac{\partial P}{\partial m} = \frac{g}{A} = \frac{10.0}{0.0001} = 100000\left[m/s^2/m^2\right]$$

Contribution to $u(y)$;

$$|c_i|u(x_i) = |c_m|u(\overline{m}) = 100000 \times 0.00894 = 894[Pa]$$

Budget sheet

Input quantities	Standard uncertainty $u(x_i)$	Evaluation method	Sensitivity coefficients c_i	Contribution to $u(y)$ $\lvert c_i \rvert u(x_i)$
Mass: m	$u(\overline{m}) = 0.00894[kg]$	Type A (five times measurements)	$100000\left[\frac{m/s^2}{m^2}\right]$	894[Pa]
Area: A				

Budget sheet

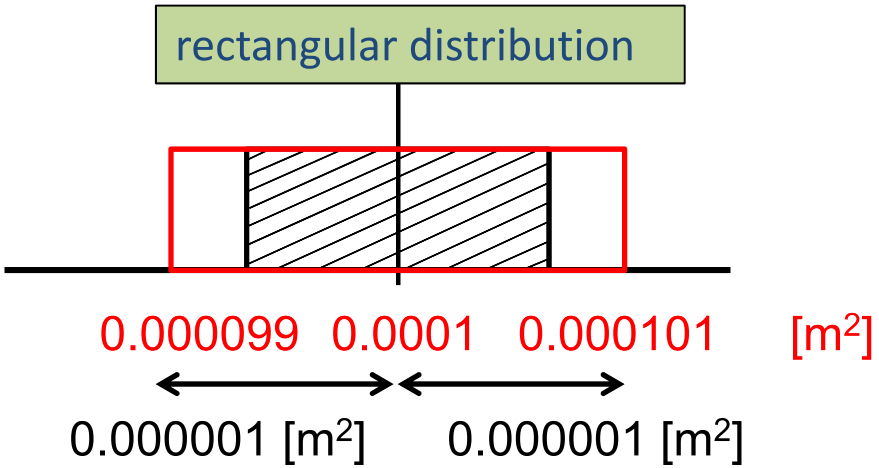
Input quantities	Standard uncertainty $u(x_i)$	Evaluation method	Sensitivity coefficients c_i	Contribution to $u(y)$ $\lvert c_i \rvert u(x_i)$
Mass: m	$u(\overline{m}) = 0.00894[kg]$	Type A (five times measurements)	$100000\left[\frac{m/s^2}{m^2}\right]$	894[Pa]
Area: A	?	?		

(2)What is the uncertainty of generated pressure?

(b)A

The area of the piston "A" is 0.0001m^2±0.000001m^2 (1cm^2±0.1cm^2) according to the manufacture's specification.

→ Type B evaluation



Standard uncertainty of A;

$$u(A) = \frac{0.000001}{\sqrt{3}} = 5.77 \times 10^{-7} [m^2]$$

Budget sheet

Input quantities	**Standard uncertainty** $u(x_i)$	**Evaluation method**	**Sensitivity coefficients** c_i	**Contribution to** $u(y)$ $\lvert c_i \rvert u(x_i)$
Mass: m	$u(\overline{m}) = 0.00894[kg]$	Type A (five times measurements)	$100000\left[\frac{m/s^2}{m^2}\right]$	894[Pa]
Area: A	$u(A) = 5.77\times10^{-7}[m^2]$	Type B (manufacture's specification)		

Budget sheet

Input quantities	**Standard uncertainty** $u(x_i)$	**Evaluation method**	**Sensitivity coefficients** c_i	**Contribution to** $u(y)$ $\lvert c_i \rvert u(x_i)$
Mass: m	$u(\overline{m}) = 0.00894[kg]$	Type A (five times measurements)	$100000\left[\frac{m/s^2}{m^2}\right]$	894[Pa]
Area: A	$u(A) = 5.77\times10^{-7}[m^2]$	Type B (manufacture's specification)	?	?

(2)What is the uncertainty of generated pressure?

Sensitivity coefficients;

$$c_i = \frac{\partial f}{\partial x_i}$$

$$\overline{m} = 1.00[kg], g = 10.0[m/s^2], A = 0.0001[m^2]$$

$$c_A = \frac{\partial P}{\partial A} = -\frac{mg}{A^2} = -\frac{1.00 \times 10.0}{0.0001^2} = -1.00 \times 10^9 \left[\frac{kg \times m/s^2}{m^4} \right]$$

Contribution to $u(y)$;

$$|c_i| u(x_i) = |c_A| u(A) = \left| -1.00 \times 10^9 \right| \times 5.77 \times 10^{-7} = 577[Pa]$$

Budget sheet

Input quantities	**Standard uncertainty** $u(x_i)$	**Evaluation method**	**Sensitivity coefficients** c_i	**Contribution to** $u(y)$ $\lvert c_i \rvert u(x_i)$
Mass: m	$u(\overline{m}) = 0.00894[kg]$	Type A (five times measurements)	$100000\left[\frac{m/s^2}{m^2}\right]$	894[Pa]
Area: A	$u(A) = 5.77\times10^{-7}[m^2]$	Type B (manufacture's specification)	$-1.00\times10^{9}\left[\frac{kg\times m/s^2}{m^4}\right]$	577[Pa]

Budget sheet

Input quantities	Standard uncertainty $u(x_i)$	Evaluation method	Sensitivity coefficients c_i	Contribution to $u(y)$ $\lvert c_i \rvert u(x_i)$
Mass: m	$u(\overline{m}) = 0.00894[kg]$	Type A (five times measurements)	$100000\left[\dfrac{m/s^2}{m^2}\right]$	894[Pa]
Area: A	$u(A) = 5.77\times10^{-7}[m^2]$	Type B (manufacture's specification)	$-1.00\times10^{9}\left[\dfrac{kg\times m/s^2}{m^4}\right]$	577[Pa]

Next:

Combined standard uncertainty ?

Combined standard uncertainty;

Square and add !

$$u_c(P) = \sqrt{[c_i u(x_i)]^2}$$

$$= \sqrt{\left[c_m u(\overline{m})\right]^2 + [c_A u(A)]^2}$$

$$= \sqrt{894^2 + 577^2}$$

$$\cong 1065[Pa]$$

Budget sheet

Input quantities	Standard uncertainty $u(x_i)$	Evaluation method	Sensitivity coefficients c_i	Contribution to $u(y)$ $\lvert c_i \rvert u(x_i)$
Mass: m	$u(\overline{m}) = 0.00894[kg]$	Type A (five times measurements)	$100000\left[\frac{m/s^2}{m^2}\right]$	894[Pa]
Area: A	$u(A) = 0.00577[m^2]$	Type B (manufactures specification)	$-1\times10^{-9}\left[\frac{kg\times m/s^2}{m^4}\right]$	577[Pa]
Combined standard uncertainty	$u_c(P) = \sqrt{[c_i u(x_i)]^2}$			$1065 \cong 1.1\times10^3[Pa]$

Expression of uncertainty

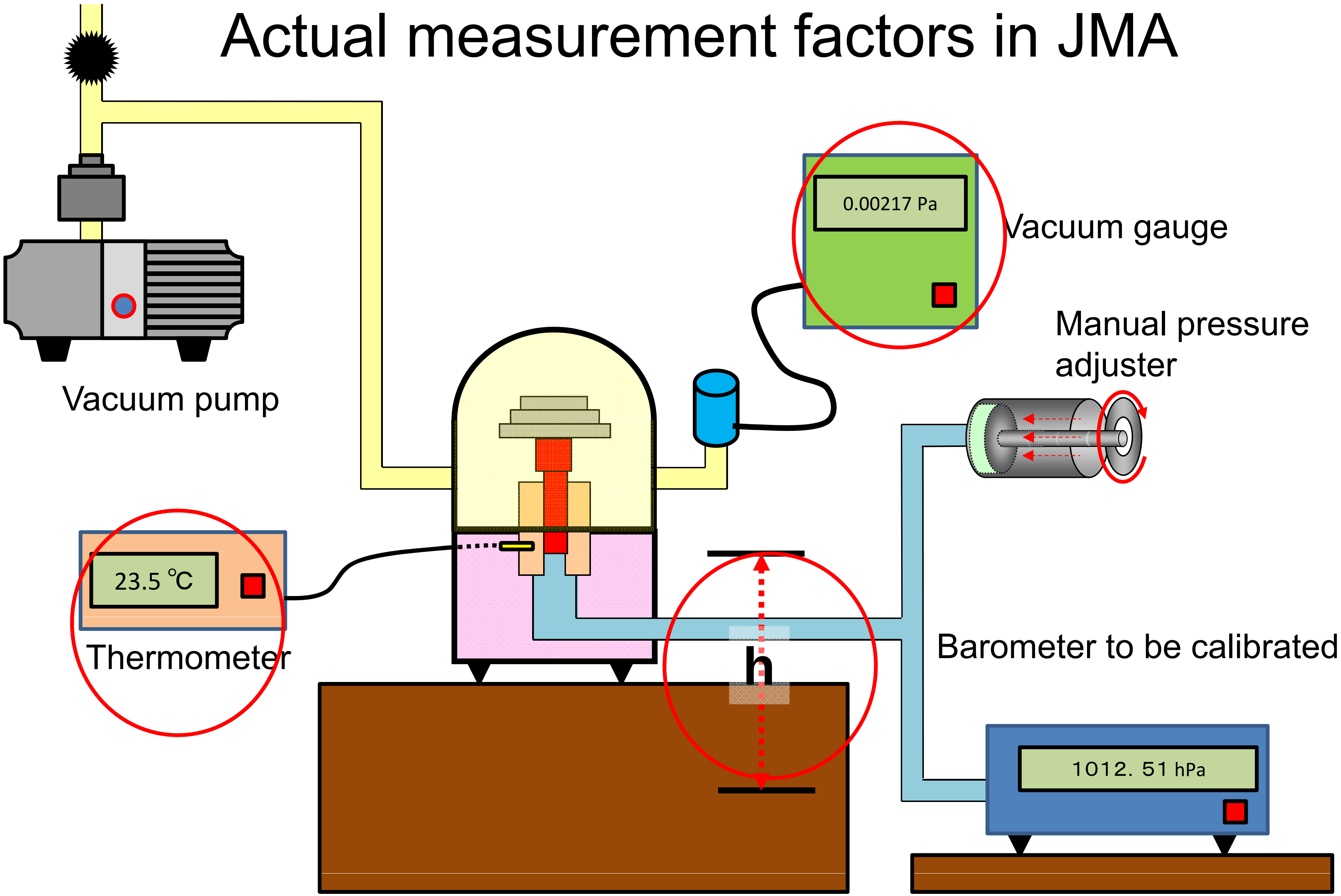
(1) Combined standard uncertainty:

"P=1000hPa, with (a combined standard uncertainty) $u_c(P)$=11hPa."

(2) Expanded uncertainty:

"P=1000±22hPa, where the number following the symbol ± is the numerical value of the expanded uncertainty corresponding (a coverage factor) *k*=2."

Actual measurement factors in JMA



Mascot of JMA ”Harerun”

気象庁